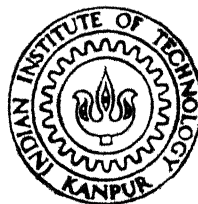# SUPPRESSION OF MACHINE TOOL CHATTER BY PASSIVE DAMPING

By

KALLOL BANDYOPADHYAY

DEPARTMENT OF MECHANICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

APRIL, 1991

SUPPRESSION OF MACHINE TOOL CHATTER
BY PASSIVE DAMPING

A Thesis Submitted

In Partial Fulfillment of The Requirements

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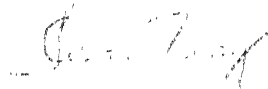## MASTER OF TECHNOLOGY

by

KALLOL BANDYOPADHYAY

to the

DEPARTMENT OF MECHANICAL ENGINEERING
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APRIL , 1991

CERTIFICATE

It is certified that the work contained in the thesis entitled " SUPPRESSION OF MACHINE TOOL CHATTER BY PASSIVE DAMPING ", by Kallol Bandyopadhyay has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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ABSTRACT

Chatter in machine tools results from a loss of stability in the cutting process due to introduction of negative damping coefficient through an interaction of system parameters and/or directional effects of modal receptances. The need to suppress chatter arises from its detrimental effects on surface finish, productivity, and machine tool life. In the present work, a simple and effective method has been suggested to improve the cutting performance by incorporating a passive-type absorber in the tangential direction. The absorber consists of a spring whose stiffness can be varied to achieve the maximum effectiveness depending upon the cutting condition. Theoretical analysis based on dynamic vibration absorber principles shows that it improves the stability of the cutting process. This requires proper tuning of the absorber w.r.t. the main system, which, otherwise will result in adverse effects on the stability limit. The effects of the various parameters of the cutting process and the absorber on the stability limit have been looked into. Experimental investigation carried out on a conventional centre lathe shows that the present system is capable of improving the overall stability up to 1.5 times within the range of experimental values of the cutting process. Lack of identical values of the system parameters prevented the comparison of the experimental and theoretical results. However, it is encouraging to note that the experimental results tend to follow the nature as predicted by the theory

ACKNOWLEDGEMENTS

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NOMENCLATURE

α_c = clearance angle (deg.)

γ_o = preset rake angle (deg.)

γ = instantaneous rake angle (deg.)

s_o = preset feed (mm/rev.)

s = instantaneous feed (mm/rev)

v_s = preset cutting speed (m/min)

v = instantaneous cutting speed (m/min)

η = cutting force ratio

ε = chip contraction factor

K = chip thickness coefficient

Δy = small movement of tool holder along thrust direction (mm)

Δz_1 = small movement of tool holder along the tangential direction (mm)

Δz_2 = small movement of absorber along the tangential direction (mm)

ΔF_y = variations in the thrust cutting force (N)

ΔF_z = variations in the tangential cutting force (N)

T_z = lag time constant under steady state cutting in the z direction (sec)

T_y = lag time constant under steady-state cutting in the y direction (sec)

m_t = mass of tool holder (kg)

m_{tz} = mass of absorber (kg)

c_{tz1} = stiffness of tool holder in the tangential direction (N/mm)

c_{ty} = stiffness of tool holder in the thrust direction (N/mm)

b_{tz1} = damping coefficient of the tool holder in the tangential direction (N.s/mm)

b_{tz2} = damping coefficient of the absorber (N.s/mm)

A_y = amplitude of vibrations of the tool holder in the thrust direction (mm)

A_{z1} = amplitude of vibrations of the tool holder in the tangential directions (mm)

A_{z2} = amplitude of vibrations of the absorber (mm)

A_{ze} = overall vibrations amplitude in the tangential direction (mm)

A_{Fy} = amplitude of thrust cutting force variations (N)

A_{Fz} = amplitude of tangential cutting for variations (N)

ω = chatter frequency (rad/s)

ω_{z1} = natural frequency of tool holder in the tangential direction (rad/sec)

w_{z2} = natural frequency of absorber (rad/sec)

ω_y = natural frequency of tool holder in the thrust direction (rad/sec)

g = ω/ω_{z1}

f = tuning ratio = ω_{z2}/ω_{z1}

p = b_{tz2}/b_{tz1}

μ_1 = mass ratio = m_{tz}/m_t

ϕ_1 = lag of ΔF_z w.r.t. Δy (deg.)

ϕ_2 = lag of ΔF_y w.r.t. ΔF_z (deg.)

ϕ_3 = lag of ΔF_y w.r.t. Δy (deg.)

ϕ_{Fz} = lag of Δz w.r.t. ΔF_z (deg)

Self-excited vibrations or chatter as it is usually called differs from the other two forms of vibrations in the respect that, it is not induced by external periodic forces, but the forces that bring it into being and maintain it are generated by the cutting process itself. Basically chatter in machine-tools may arise through either or both of the following causes:

a) introduction of negative damping coefficient either through the cutting process itself or through an interaction of system parameters.

b) directional effects of modal receptances.

Chatter is thus a self-sustaining process which draws energy from an extraneous source by its own periodic motion and hence always occurs at a frequency close to the natural frequency of the undamped system. The mechanism of chatter can be divided into three groups:

a) primary chatter

b) regenerative chatter

c) mode coupling.

Under practical machining conditions, the mechanisms described above do not remain independent of each other, but on the contrary, an associated together and all the three mechanisms occur simultaneously in varying degrees. Of these, it is the regenerative chatter that is the most likely to dominate. Thus while considering methods of controlling chatter, one must always try to develop an unified theory capable of explaining all the phenomenon observed from the view point of dynamic cutting

CHAPTER 1

1.1 INTRODUCTION

In the modern day of manufacturing accuracy and reliability of the products are gradually becoming prominent features. In order to achieve higher standards of productivity and accuracy one has thus to take into account the static as well as the dynamic characteristics of the machine-tool-workpiece system. If there is any relative movement between the tool and the work then obviously the performance of the machine cannot be called satisfactory. Moreover, machine tool vibration has detrimental effect upon the tool life which in turn leads to lower productivity and to higher costs of production. During operations the machine-tool is subjected to both static as well as dynamic loads, the latter having period and/or impact characteristics associated with the cutting process. These loads may act through the following ways:

a) dynamic behaviour caused entirely by the load acting during the action of the load (forced vibrations);

b) dynamic behavior initiated by the load but persisting after the load has ceased to act (free vibrations);

c) dynamic behavior initiated through an interaction between the structure and the cutting process (self-excited vibrations).

Of these, the first two types of vibrations, i.e. free and forced vibrations do not present any new problems and can be suppressed by the prevailing techniques. Once the cause of vibration is identified, it is always possible to find methods to eliminate them.

Self-excited vibrations or chatter as it is usually called differs from the other two forms of vibrations in the respect that, it is not induced by external periodic forces, but the forces that bring it into being and maintain it are generated by the cutting process itself. Basically chatter in machine-tools may arise through either or both of the following causes:

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mechanics as well as machine-tool structure dynamics. Furthermore, since chatter initiates as a result of the cutting process itself, it becomes necessary to look into the various influences of a machining process (machine-tool-workpiece combination and cutting process) upon stability conditions while trying to implement steps to avoid chatter vibrations.

1.2 REVIEW OF PREVIOUS WORK

Chatter in machining impedes the improvement of cutting accuracy and high speed machining. Thus it becomes important to review the causes of chatter as well as the methods to suppress it.

1.2.a Causes of Chatter:

As stated in Section (1.1), the causes of chatter may be classified into three groups.

The primary chatter is caused by a) the falling characteristics of the cutting force but mainly b) the time lag between the cutting force variations and the chip-thickness variations.

In the so called velocity principle, it is assumed that a component of the cutting force depends on and being in phases with the velocity of vibration provides the energy for initiating and sustaining vibrations.

The belt-friction model (Van der Pol model) provides the mechanical basis to explain the physical causes of chatter and the

limited amplitude mechanism. Doi and Kato from their previous work concluded that chatter in its character resembled that of frictional vibrations. In the Van der Pol model, the kinetic friction force is assumed to be a function of the cutting velocity. If the slope of this frictional force is negative, instability arises and vibrational amplitudes begin to grow once it overcomes the positive damping of the system.

Arnold [1] in his single degree of freedom system analysed the chatter phenomenon by assuming the cutting force to be a function of the instantaneous cutting velocity, feed and rake angle. However, as the amplitudes of vibrations grow up, system nonlinearities begin to exert their influences, and thus prevent the amplitudes being build up beyond a limiting amplitude. However, it is doubtful whether the negative damping introduced by the falling characteristics is capable of overcoming the positive damping of the system, because as pointed out by Tobius [2], the amount of negative damping introduced is not only proportional to the negative slope of the curve but also dependent upon the dynamic behavior of the machine tool.

Cook [4] analysed the chatter phenomenon using a two-degree of freedom system. He argued that with an increase in the chip velocity relative to the tool, the temperature at the chip-tool interface increases and hence the friction force decreases. The normal force exerted on the chip tool interface however is incapable of directly initiating vibrations. He argued that, if the velocity of chip relative to the tool rake face varies in any way, then it is possible to have a net energy input into the

system which can initiate and sustain vibrations in a direction parallel to the face. Similar conclusions can also be drawn along the tangential direction due to the rubbing of the flank face with the work. Thus, the energy input to the system causing vibrations can be the sliding of the chip along the tool rake face as well as rubbing along the flank face. Amplitude limitations occur due to plastic deformations as a result of interference along the flank face. Amplitude limitations can also occur due to the chip leaving the tool surface during a part of the cycle. He also concluded that the shearing process damps vibrational motion parallel to the shear plane.

Wu and Liu [5,6] assumed the dynamic cutting as a two degree-of-freedom system with two types of forces in action: a) cutting forces exerted on tool rake face and b) ploughing forces on the tool nose region. They argued that the geometric configuration of the cutting process under dynamic cutting do not remain constant, but fluctuates since the frictional coefficient also fluctuates dynamically according to instantaneous cutting conditions. In their model, the chip material moves over the tool rake with a variable speed while exerting a normal force upon it. The chip forming mechanism controls the chip velocity and the normal force thus inducing a variable frictional force between the chip and the tool-block system. Instability arises in the same way as stated earlier but the amplitude limitations occur due to the generation of an additional damping in the tool nose region by interference along tool-work interface as was first pointed out by

Cook [4]. The chip forming mechanism, however, is controlled by the instantaneous cutting conditions. Under dynamic conditions, any change in the cutting conditions leads to changes in the values of the normal force, relative chip-sliding velocity and frictional force instantaneously, thus affecting the system behavior.

The model proposed by Doi and Kato [3] is important noty only from the viewpoint of chatter theory but also from the viewpoint of the cutting theory as well. They showed that during cutting, the horizontal and vertical cutting force components lag behind the chip thickness variation. Doi and Kato regarded this delay in cutting force variations as a fundamental effect and suggested that this delay in forces with respect to chip thickness variation is the cause of chatter. The delay period becomes larger with increasing feed and wedge angle but decreases with increasing cutting speed [7,8].

Elyasberg [8,9,10] suggested that the cause of forces lagging in metal cutting are in fact specific characteristics of the cutting process. During cutting, the tool edge does not continuously participate in the cutting process, rather its role is restricted to the formation of cracks in the metal layer being removed. This crack formation while cutting metals, which in addition to having elasticity, have brittleness or ductility and work hardening tendencies, is unavoidable. Elyasberg suggest that this crack formation causes the variation in vertical cutting force to lag behind the chip-thickness variation. Similarly, the

frictional force along the rake face lags the tangential force by the time necessary for the chip to get from the shear plane to the point on the tool face where the stress between the chip and the tool is greatest. This leads to a phase difference between the oscillatory components of the cutting force and the tool oscillations.

Tobias [2] pointed that this delay can be explained by considering that the cutting force variations depend upon the rate of penetration (i.e. instantaneous feed rate). However, the variation of phase lag as a function of wedge angle cannot be explained in the same way.

Whereas in primary chatter, there is no interaction between the vibratory motion of the system and any undulations produced in the preceeding revolution of the workpiece, in regenerative chatter, the cutting conditions are such that the vibratory system is subjected to the effect of undulations produced in the preceeding revolution. The undulations increase with subsequent revolutions and the phase of undulations always lag the preceeding undulation. Because of this phase lag, the area being cut in an approaching stroke will be less than that being cut in the recess stroke so that the cutting force in latter is greater resulting in an energy being supplied to the system. The stability boundary in regenerative chatter decreases with increasing time lag and overlap factor. But the phase lag of undulations is determined only by the time lag of cutting force and is independent of the

The concept of overlap factor in regenerative chatter was introduced by Gurney and Tobias [13] to define the portion of the previous cut that overlaps the present cut. The overlap factor can also be used to account for the geometrical effects of rounding at the tool cutting edge and of the clearance angle. Both of these effects tend to smear the machined surface and thereby reduce vibrational amplitudes. Under practical conditions it is difficult to precisely define the overlap factor, but it always lies between zero and one. An overlap factor of unity is the most critical value from the viewpoint of chatter.

The mode coupling effect occurs when the principal modes of the vibratory systems are closely matched. The cutting force acting along one directional axis may cause a tool motion along another direction resulting in instability. In mode coupling, the cutting edge describes an approximately elliptical path. Tlusty [14] showed that if the tool describes this elliptical path, it is possible to supply energy into the system capable of exciting and maintaining vibrations. Stability of the system decreases with increasing matching between the two principal modes of vibrations. Thus, the degree of stability against mode coupling may be improved not only by increasing the stiffness and damping of the system but also by changing the mutual tuning of the modes as regards the direction of orientation. This helps in increasing stability without substantial change in the weight of the machine tool.

1.2.b Elimination of Chatter:

The measures taken to reduce chatter may be classified in the following way [26]:

measures against chatter

```
Constructional    influencing the interaction       Changes in the
                  of process and structure           cutting process

enhancement       variation of spindle               better cutting
of dynamic        speed during cutting               parameter
stiffness


higher            damping
static
stiffness


        active                          passive
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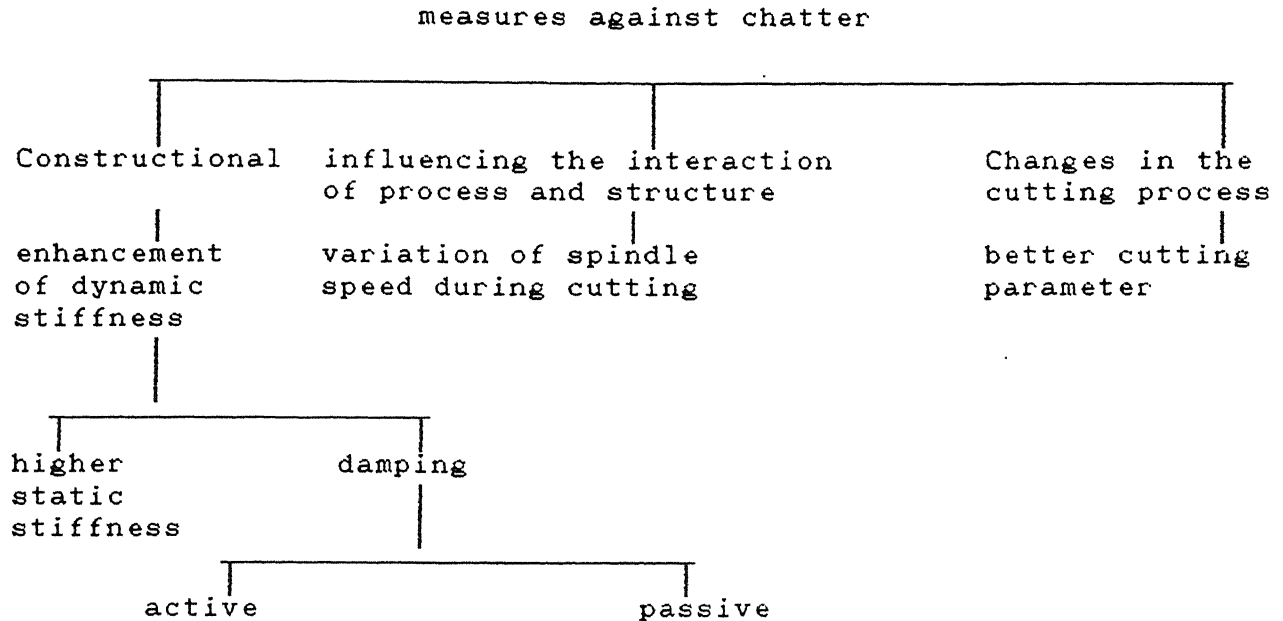
The changes in cutting process that help in reducing chatter in machine tools are,

a) minimising the clearance angles;

b) negative rake angles;

c) increasing the feed rate (i.e. penetration rate)

d) choice of high or low cutting speed to avoid the minimum
    stability point;

e) on multiple cutting edged tools, choice of our uneven
    number of cutting edges.

However, these methods cannot be called satisfactory since one has also to consider the effects of these changes on the cutting process itself. Minimising the clearance angle may help

in reducing chatter, but increases the wear rate of the tool due to increased rubbing with the work. Negative rake angle increases the cutting force which in turn necessitates introducing better clamping conditions for the job and the tool. Moreover, increased cutting forces increase the wear rate of the tool and at the same time reduce the productivity by lowering the allowable cutting speed. While the choice of low speed cutting is governed by the economic viewpoint of machining, the choice of high cutting speeds will be governed by the machine-tool-work combination. Thus, one can see the complexities that has to be taken into account while implementing changes in the cutting process.

A better way to improve stability can be provided by influencing the interaction of the process and structure in such a way so that the resultant cutting force is perpendicular to the direction of highest dynamic flexibility of the machine. Thusty [14] utilised this mutual orientation of the cutting process and the structure on centre lathes to obtain significant improvement in stability. He fixed a plate to the cross-slide in such a way so that the tool could be fixed at various angles to the horizontal. From polar curves be showed that the usual position of the tool post corresponds to the worst or almost the worst stability conditions.

Continuous varition of spindle speeds is another method of suppressing chatter through the mutual interaction process [15,16,17]. The stabilising effect from varying spindle speed result because [17];

a) the phase angle between the vibrations of the tool and the wave on the workpiece surface will vary continuously and the "critical" phase angle leading to instability will be rarely achieved;

b) the waves on the workpiece surface will be removed at a speed different from that at which they more left by the tool one revolution previously, and, hence, the tool will not be excited at a constant frequency near the natural frequency of the system but at a continuously varying frequency.

Although variations in spindle speeds result in improved stability but the improvements are not sufficiently great to justify the expenses of the additional equipment necessary to vary the spindle speeds continuously. Moreover, the surface finish at increased widths of cut (due to improved stability conditions) is poor due to the presence of the transient vibrations and, hence, the advantage of improved stability cannot be taken when the finest finish is required.

Of all the methods available, it is the enhancement of the dynamic stiffness through constructional efforts which is most effective. Rather than increasing the static stiffness, introducing damping either through active or passive means helps to improve stability greatly. The main sources of damping in machine tools are the structural damping in the machine tool itself and frictional damping in which energy is dissipated due to the rubbing of the surfaces of two elements at the interface. Friction or slip damping can be increased by using welded joints.

Friction damping however cannot be used for large displacements if the elements are located in series. Some increase in damping is possible in cases where the elements are located parallel to the force loop where large displacements can be allowed. But in all cases, although a comparative increase in damping is possible to attain, it does not provide enough energy dissipation to give a sufficient effect [21].

Ryzkhov designed an impact damper in which a mass is tuned to the natural frequency of the tool. When the tool vibrates, the damper is likewise thrown into vibrations, thus producing impacts with the tool holder resulting in the dissipation of energy. The system seems to be suitable for cases where the tool has a long overhang [2]. The drawback of this impact damper include the dependence on gravitational influences as well as on the spring suspension of the impacting mass.

The dynamic absorbers improve the response of the primary system by adding an auxilliary mass to the main system. This shifts the resonant frequency of the mechanical system away from the operating frequency of the vibratory force. Whereas in passive system, tuning of the secondary mass with the primary mass is very important since mismatching produces adverse results, active dampers can work over a wide range of frequencies as they have the advantage of not having any servo-instability and since the matching with the primary system is not critical. In active dampers, a control system acts on the signal derived from the main system motion and activates a force generator to produce the

inertial reaction.

Cowley and Boyle [21] presented an analysis of active dampers based on electrodynamic principles. The electrodynamic generator produces a force proportional to the instantaneous signal produced from the transducer mounted on the structure. They showed that the electrodynamic vibrator controlled by velocity feedback was capable of providing additional damping proportional to the feedback gain.

Sata and Inamura [22] developed a method to predict and prevent the growth of chatter based on fundamental knowledge of chatter mechanism and by using analytical methods for evaluating the dynamic characteristics of the cutting process. The method to suppress chatter lies in trying to find out an effective and simple way to separate the stiffness transfer function locus of the machine tool structure from the stiffness transfer function locus of the cutting process which come into contact during regenerative chatter. To reduce the radius of the stiffness circle of the cutting process they considered the variation of the feed rate, depth of cut, nose radius and the side cutting edge of the tool. The system developed can choose the most effective parameter to suppres chatter by modifying the cutting condition.

Seto [23] proposed a new design method for a variable stiffness type dynamic absorber with viscous damping to increase the cutting performance of machine tools with long overhung rams like in boring bars and vertical lathes. The variable stiffness type dynamic absorber has a controllable frequency turner for

tuning the absorber to the maximum efficiency over the interested frequency range. The stiffness of the spring was determined from the condition of optimum tuning. Experiments conducted show that the stability can be improved by fifteen times with the absorber.

Kim and Ha [24] suppressed chatter by attaching an optimum-designed viscoelastic dynamic damper to the tool post of a lathe. Since the chatter frequency varies depending upon the location of the carriage, the prestrain of the viscoelastic element was adjusted for optimum tuning at different locations. The viscoelastic damper is relatively simple to fabricate. The compliance of the tool post was efficiently reduced by readjusting the prestrain of the viscoelastic element.

Rivin and Hongling Kang [25] tried to improve the machining of slender parts by tuned dynamic stiffness of the cutting tool. They showed that the system stability can be improved by adding a damping element in series with the cutting tool and through a judicious choice of the tool stiffness, although, this reduces the actual stiffness of the cutting tool. When the tool stiffness is intentionally reduced, it becomes necessary to analyze the system stability using a two degree of freedom system. At optimal damping, the effective cutting stiffness increases leading to higher stability limit. They showed that the mean vibrations between the job and the tool decreased in spite of the fact, that the amplitudes of vibrations of the tool increased by about three times in comparison to that of the original solid tool.

1.3 OBJECTIVES OF THE PRESENT WORK

The present work is in the field of machine tool chatter. Since chatter in machine tools reduces the quality of surface produced and productivity, it bnecomes necessary to devise methods to reduce chatter. The present work attempts to design a passive-type dynamic vibration absorber to control chatter aiming to achieve better dimensional accuracy and surface finish.

Under steady state cutting, the relative displacement between the tool and the work resulting from the deformation of the tool-work-machine system does not change and hence, there is no distortion in the form accuracy of the work. However in practical cases, steady state cutting conditions never exists. The relative displacements under actual machining conditions vary due to:

a) variation in the value of the resulting compliance, and

b) variation in the values of the cutting forces.

In steady state cutting, the cutting forces depend on the nature of the tool-work interface, the feed rate, cutting speed and depth of cut. Under actual conditions, the parameters stated above do not remain constant but vary about a mean value in a random manner. Due to this vibratory nature of the cutting forces on the tool, the resulting compliance also varies depending upon the frequencies of vibrations. And since, dimensional in -accuracies are a direct consequence of the variation of the resulting compliances, it becomes necessary to reduce the latter through simple and effective means. While trying to reduce the

compliance, it is necessary to know not only the relationship between the amplitudes of various forces and displacements but also their phase relationships. In a machine tool system stability can be improved by decreasing compliance in one dirction keeping the compliance same in the other directions, and in some cases, even by increasing the compliance in a particular direction [14]. Furthermore, since chatter occurs due to the phase lag of the cutting forces behind chip thickness variations [3, 8, 9, 10, 11] any effort to reduce this lag will introduce positive damping into the system and hence increase the overall stability limit. It is essentially with this view in mind that the present work is attempted. In the present work, the technique proposed by Seto [23] has been extended to the case of turning in a conventional centre lathe. Higher cutting stiffness and hence higher stability limits can be obtained by incorporating absorbers to the tool post. The addition of absorbers not only reduces the compliance of the main system, but also through a proper choice of parameters it is possible to increase the effective damping in the main system leading to improved machine tool stability.

Thus the objective of the present work may be stated in brief as:

a)   to design a passive type dynamic vibration absorber for lathe tools to improve stability;

b) to carry out theoretical analysis based on established chatter theories to determine the effect of various parameters so that for a given cutting condition maximum stability can be achieved;

c) experimental investigation of the proposed system.
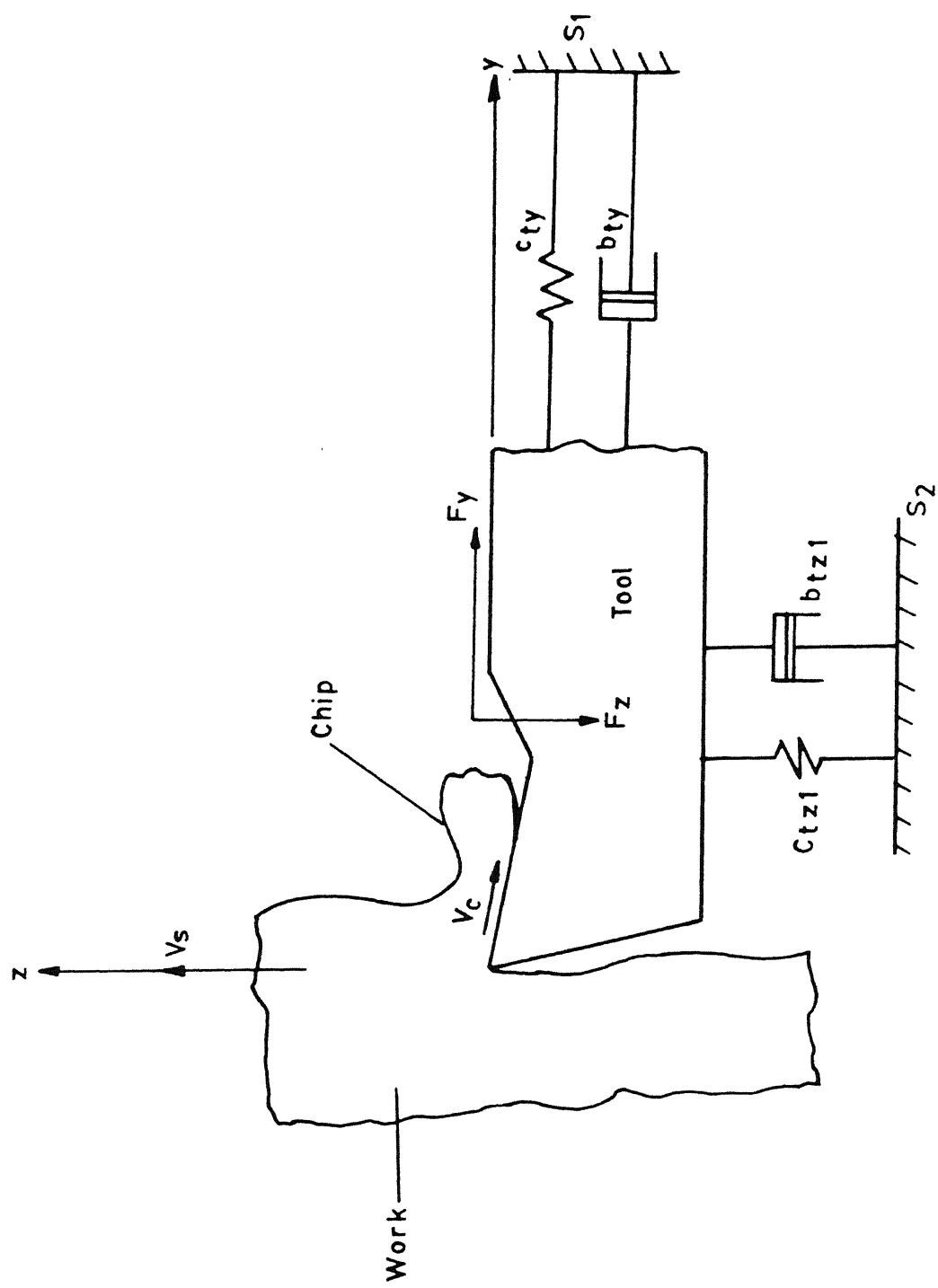
Fig. 2.1 Vibration model of a dynamic cutting process

CHAPTER 2

THEORETICAL ANALYSIS

2.1 STABILITY OF THE TWO-DEGREE OF FREEDOM SYSTEM

The analysis has been carried out under the following assumptions:

a)   The cutting is orthogonal;

b)   The tool is sharp;

c)   There is no rubbing along the clearance face;

d)   The work is infinitly stiff;

e)   The tool is infinitly stiff in the x-direction;

f)   There is no built-up edge formation.

In lathe operations under conditions of restricted orthogonal cutting (orthogonal system of the first kind) the resultant cutting force can be broken down into three components:

F_z -- tangential cutting force tangent to the surface of cut and coinciding with the primary cutting motion direction.

F_y -- radial cutting force acting in a horizontal plane perpendicular to the axis of the work.

F_z -- axial or feed force acting parallel to the axis of the work opposing the feed motion.

Following assumption (e) the vibration model of the dynamic cutting process can be represented as in Fig.(2.1).Positive y is assumed to be in the direction in which the depth of cut is reduced.

Fig. 2.4 Chip thickness variation under dynamic condition

Fig. 2.2 Force diagram of orthogonal cutting under dynamic condition
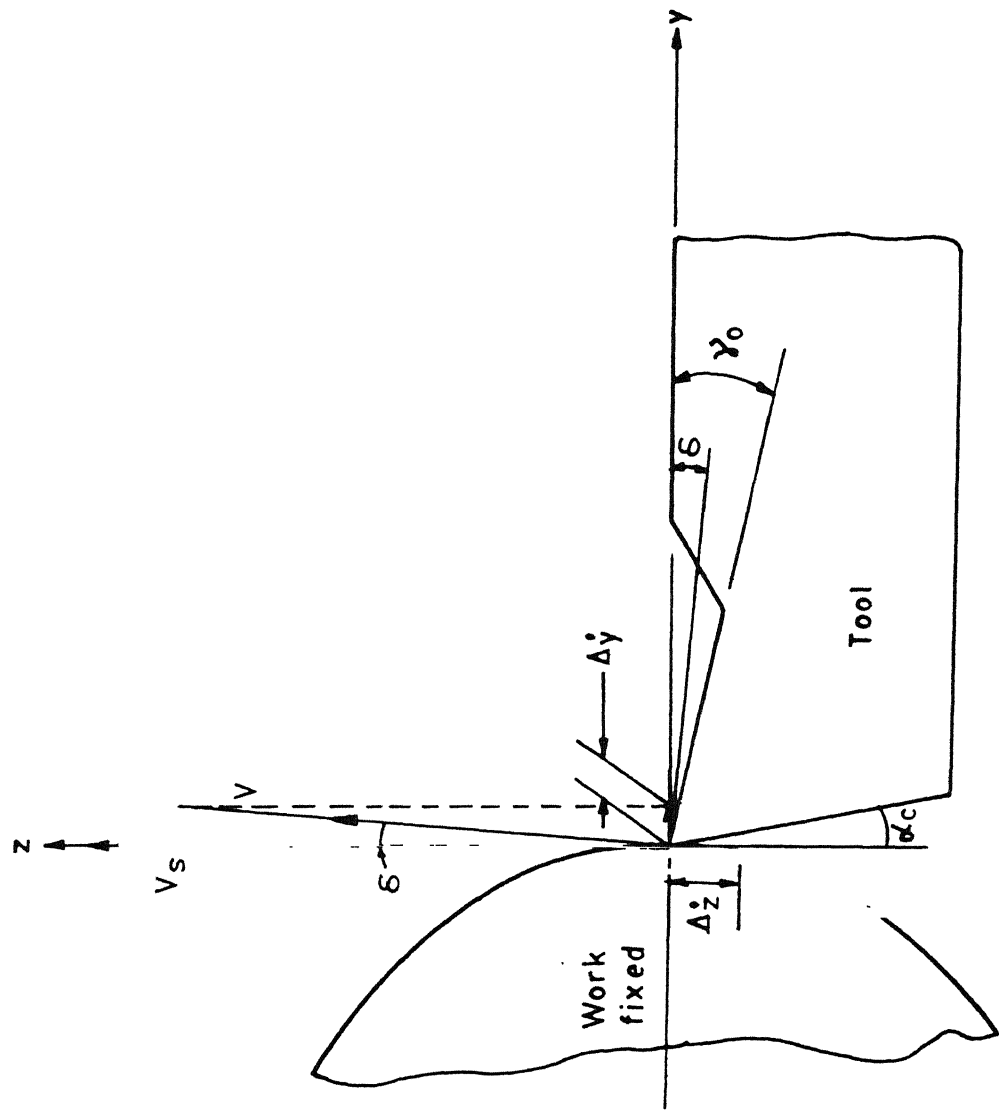
Fig.2.3 Velocity conditions at the cutting edge during vibrations

In the figure a flexible mass (tool holder) m_t is attached to support S1 and S2 through springs c_{ty} and c_{tz1} and dampers b_{ty} and b_{tz1}. The chip moves over the tool rake face pressed against it by a normal force F_N. This normal force, thus, induces a frictional force F_f between the chip and tool through the sliding process.

Under steady-state conditions, the tool tip stays at the origin of the coordinate system while under dynamic cutting, the tool tip fluctuates about the origin. Similarly the surface end of the shear plane remains stationary under steady-state conditions but fluctuates under dynamic conditions. The relationship between the various forces under dynamic cutting becomes dependent upon the instantaneous conditions of the parameters of cutting and the relationship between the various forces has been shown in Fig. (2.2). The condition at the tool tip at any instant of time is shown in Fig. (2.3). The resultant cutting velocity in the tangential direction at any instance of time $v = v_s - \Delta \dot{z}$. The rake angle during vibration

$$\gamma = \gamma_o - \tan^{-1} \left(\frac{\Delta \dot{y}}{v_s - \Delta \dot{z}} \right) \tag{1}$$

From Fig. (2.2)

$$F_y = \eta \, F_z \tag{2}$$

where, F_y = thrust force

F_z = tangential force

η = cutting force ratio

$$= \frac{F_y}{F_z}$$

Under dynamic cutting, the chip thickness ratio η becomes dependent upon instantaneous cutting condition since the friction angle and rake angle an also take instantaneous values. Using the formula in [26] under no built-up edge conditions:

$$sv^{0.9} \geq 0.61 + 1.2 \, Sin\gamma \qquad (3)$$

$$\eta = 0.75 - 0.0.87 \, Sin\,\gamma - (0.098+0.072 \, Sin\gamma) \left[\frac{ln\,(sv^{0.9})-6.76}{s^{0.1}} \right]$$

and chip contraction factor

$$\varepsilon = 0.56 Sin\gamma + (.102-0.025 Sin\gamma) \left[\frac{ln\,(sv^{0.9})+0.15}{s^{0.1}} \right]$$
$$(5)$$

where, s = instantaneous chip thickness

$$= s_o + \Delta y - \Delta y_o$$

v = instantaneous cutting velocity in the tangential dirdection

$$= v_s - \Delta\dot{z}$$

γ = rake angle = $\gamma_o - tan^{-1} \left(\frac{\Delta \dot{y}}{v_s - \Delta\dot{z}} \right)$

Under dynamic cutting conditions, there is no immediate corresponding changes in tangential and thrust cutting forces due to tool movements. The changed elastic forces are balanced by inertia forces and internal frictional forces which take certain time to reach their corresponding full values. The cause of this delay can be related to the deformation characteristics of the sheared metal layer [3,8]. During cutting, under conditions of continuous chip production, the shear strength of the chip depends upon the compressive strength normal to the plane of shear. The magnitude of this compressive stress depends upon the frictional characteristics at the chip tool interface [3]. By the time the balancing inertial and frictional forces reach their full value the tool traverses certain lengths of cut relative to the work surface. These lengths can be considered as constant for given metal and cutting rates. Thus, the lag time of cutting forces become variable under dynamic cutting but remain constant under steady-state cutting. These lag constants increase with increasing feed and wedge angle but decrease with increasing cutting speed [3,8]. Since, the tangential cutting force variations lag the chip thickness variation and the thrust force variation lag the variations in tangential cutting forces therefore, one can write

$$\Delta F_z(t) = -K\Delta y (t - h_1) \qquad (6)$$

and

$$\Delta F_y(t) = \eta \Delta F_z(t - h_2) \qquad (7)$$

where K = chip thickness coefficient

          h_1 = time lag of tangential cutting force variation behind chip thickness variation

          h_2 = time lag of thrust cutting force variation behind tangential force variation

Now,

$$h_1 \approx \frac{l_{Fz}}{v_s - \Delta \dot{z}} \qquad (a)$$

and

$$h_2 \approx \frac{l'_{Fy}}{v_c + \Delta \dot{y}} \qquad (b)$$

$$(8)$$

where, l_{Fz} = lag distance covered by tool in direction z for a small disturbance along y direction.

l'_{Fy} = lag distance covered by tool in y direction for a small movement along y.

     v_c = instantaneous chip flow velocity

$$\varepsilon = \tan^{-1}\left(\frac{\Delta \dot{y}}{v_s - \Delta \dot{z}}\right) \approx \tan^{-1}(\Delta \dot{y}/v_s)$$

again

$$h_2 = \frac{l'_{Fy}}{v_c + \Delta \dot{y}}$$

$$= \frac{l'_{Fy}}{\dfrac{V\ Cos\varepsilon}{\varepsilon} + \Delta \dot{y}} \qquad \frac{l_{Fy}}{(v_s - \Delta \dot{z}) + \varepsilon_1 \Delta \dot{y}}$$

$$= \frac{l_{Fy}}{(v_s - \Delta\dot{z}) + \epsilon_1 \Delta\dot{y}}$$

where, $\epsilon_1 = \epsilon/\cos\delta$

l_{Fy} can thus be considered as the distance travelled by the tool in direction z to substitute for the lag distance l'_{Fy} along y. Now taking,

$$l_{Fz} = v_s T_z$$
$$l_{Fy} = v_s T_y \qquad (9)$$

one can write,

$$h_1 = \frac{v_s T_z}{v_s - \Delta\dot{z}} \qquad (10a)$$

$$h_2 = \frac{v_s T_y}{(v_s - \Delta\dot{z}) + \epsilon_1 \Delta\dot{y}} \qquad (10b)$$

where, T_y, T_z = lag time constant along directions y and z respectively.

From (6) and (7) expansion in a Taylor series gives,

$$\Delta F_z + h_1 \Delta \dot{F}_z + K \Delta y = 0 \qquad (11a)$$

and

$$\Delta F_y + h_2 \Delta \dot{F}_y - \eta \Delta F_z = 0 \qquad (11b)$$

Substituting the values of h_1 and h_2 from (10a) and 10(b)

$$\Delta F_z + \frac{v_s T_z \Delta \dot{F}_z}{(v_s - \Delta \dot{z})} + K \Delta y = 0 \qquad (12a)$$

$$\Delta F_y + \frac{v_s T_y \cdot \Delta F_y}{(v_s - \Delta \dot{z}) + \varepsilon_1 \Delta \dot{y}} - \eta \Delta F_z = 0 \qquad (12b)$$

from (12a)

$$(v_s - \Delta \dot{z}) \Delta F_z + v_s T_z \Delta \dot{F}_z + K \Delta y (v_s - \Delta \dot{z}) = 0$$

or, $\quad v_s (T_z \Delta \dot{F}_z + \Delta F_z + K \Delta y) = K \Delta y \Delta \dot{z} + \Delta F_z \Delta \dot{z}$

neglecting the first term on the right since $\Delta y \Delta \dot{z} \ll v_s \Delta y$, one has

$$v_s \left[T_z \Delta \dot{F}_z + \Delta F_z + K \Delta y \right] = \Delta F_z \Delta \dot{z}$$

or,

$$T_z \Delta \dot{F}_z + \Delta F_z + K \Delta y = \frac{\Delta F_z \dot{z}}{v_s}$$

in nondimensional form

$$T_z \frac{\Delta \dot{F}_z}{F_z} + \frac{\Delta F_z}{F_{zo}} + \frac{K \Delta y}{F_{zo}} = \frac{\Delta F_z \Delta \dot{z}}{v_s F_{zo}}$$

or,

$$T_z \cdot \dot{F} + F_z + \frac{\eta_o K y}{c_{ty}} = \frac{\Delta F_z \dot{z}}{v_s c_{tz1}}$$

Since,

$F_{yo} = \eta_o F_{zo}$ under steady-state conditions and

$$F_{yo} = c_{ty} y_o$$

$$F_{zo} = c_{tz} z_o \qquad (13)$$

$$T_z \dot{F}_z + F_z = -k_y y + \frac{\Delta F_z \dot{z}}{v_s c_{tz1}} \qquad (13a)$$

$$= -k_y y + T_{kz1} \dot{z}$$

where,

$$k_y = \frac{\eta K_v}{c_{ty}} \quad \text{and} \quad T_{Kz1} = \frac{\Delta F_z}{v_s c_{tz1}}$$

Again from (12b),

$$V_s \left[\Delta F_y + T_y \dot{\Delta F}_y - \eta \Delta F_z \right] = (\eta \Delta F_z - \Delta F_y)(\varepsilon_1 \dot{\Delta y} - \dot{\Delta z})$$

$$\Delta F_y + T_y \dot{\Delta F}_y - \eta \Delta F_z = \frac{(\eta \Delta F_z - \Delta F_y)(\varepsilon_1 \dot{\Delta y} - \dot{\Delta z})}{v_s}$$

in nondimensional form,

$$\frac{\Delta F_y}{Fyo} + T_y \frac{\dot{\Delta F}}{F_{yo}} - \frac{\eta \Delta F_z}{F_{yo}} = \frac{(\eta \Delta F_z - \Delta F_y)(\varepsilon_1 \dot{\Delta y} - \dot{\Delta z})}{v_s Fyo}$$

Taking $\eta \approx \eta_o$,

$$F_y + T_y \dot{F}_y - F_z = \frac{(\eta \Delta F_z - \Delta F_y)(\varepsilon_1 \Delta \dot{y} - \Delta \dot{z})}{v_s F_{yo}}$$

$$= \frac{\varepsilon_1 (\eta \Delta F_z - \Delta F_y) \Delta \dot{y}}{v_s F_{yo}} - \frac{\eta \Delta F_z (1 - \Delta F_y / \eta \Delta F_z) \Delta \dot{z}}{v_s F_{yo}}$$

$$= \frac{\varepsilon_1 (\eta \Delta F_z - \Delta F_y) \dot{y}}{v_s c_{ty}} - \frac{\Delta F_z \dot{z} (1 - \Delta F_y / \eta \Delta F_z)}{v_s c_{tz1}}$$

$$= T_{ky12} \, \dot{y} - T_{kz2} \, \dot{z} \qquad (13b)$$

where,

$$T_{ky12} = \frac{\varepsilon_1 (\eta \Delta F_z - \Delta F_y)}{v_s c_{ty}} \quad \text{and} \quad T_{kz2} = \frac{\Delta F_z (1 - \Delta F_y / \eta \Delta F_z)}{v_s \cdot c_{tz1}}$$

Adding (13a) and (13b)

$$T_z F_z + T_y \dot{F}_y + F_y + k_y y - T_{ky12} \dot{y} - T_{kz12} \, \dot{z} = 0 \qquad (14)$$

where,

$$T_{ky12} = \frac{\varepsilon_1 (\eta \Delta F_z - \Delta F_y)}{v_s \, c_{ty}}$$

$$T_{kz12} = T_{kz1} - T_{kz2} \frac{\Delta F_y}{\eta \quad v_s \, c_{tz1}}$$

Under dynamic conditions for small movements Δz and Δy the equations of motion for the y and z directions can be written as,

$$m_t \Delta \ddot{z} + b_{tz1} \Delta \dot{z} + c_{tz1} \Delta z = \Delta F_z \qquad (15a)$$

$$m_t \Delta \ddot{y} + b_{ty} \Delta \dot{y} + c_{ty} \Delta y = \Delta F_y \qquad (15b)$$

in nondimensional form,

$$T_{z2}^2 \ddot{z} + T_{z1} \dot{z} + z = F_z \qquad (16a)$$

$$T_{y2}^2 \ddot{y} + T_{y1} \dot{y} + y = F_y \qquad (16b)$$

where

$$T_{z2} = \frac{1}{\omega_{z1}} \qquad\qquad T_{y2} = \frac{1}{\omega_y}$$

$$T_{z1} = b_{tz1}/c_{tz1} \qquad\qquad T_{y1} = b_{ty}/c_{ty}$$

The equations (13a), (13b) along with (16a) and (16b) describe completely the state of the tool at any instant of time.

Equation (14) shows that the terms associated with $\dot{y}$ and $\dot{z}$ tend to introduce negative damping into the system. Thus, in order to improve the stability limit the effect of these terms have to be reduced. This effort forms the basis to find out and devise methods to design absorbers with an intention to improve the stability limit.

The characteristics equation for the system when all the terms are present can be shown to be as:

$$(E_{Fy} E_y - s T_{ky1}) (E_{fz} \cdot E_z - s T_{kz1}) + k_y (E_z - s T_{kz2}) = 0$$

(17)

where,

$$F_{Fy} = s T_y + 1$$

$$E_{Fz} = s T_z + 1$$

$$E_z = (s^2 T_{z2}^2 + s T_{z1} + 1)$$

$$E_y = (s^2 T_{y2}^2 + s T_{y1} + 1).$$

Solution of equation (17) in terms of k_y helps to find out the stability limit. However, under simplifying assumptions the basic sixth order equation can be reduced to the simplest third order system.

In the simplest third order, the basic system is considered as a single degree of freedom system capable of vibrating in the thrust direction (direction y) only and the delay of forces F_z and F_y are approximated by the overall delay of F_y w.r.t. y so that

$$T_{yz} = T_y + T_z$$

(18)

For the third order system,

$$T_{y1}^2 \ddot{y} + T_{y1} \dot{y} + y = F_y$$

(19a)

$$T_{y2} \ \dot{F}_y \ + \ F_y \ =- \ k_y y \qquad\qquad (19b)$$

Substitution of (19a) in (19b) gives on arrangement

$$T_{y2}^2 \ T_{yz} \ \dddot{y} \ + \ (T_{y2}^2 \ + \ T_{yz} \ T_{y1}) \ \ddot{y} \ + \ (T_{y1} \ + \ T_{yz}) \dot{y} \ + \ (1+k_y)y \ = \ 0$$
$$(20)$$

Under relaxed conditions, the Laplace transform gives,

$$T_{y2}^2 \ T_{yz} \ s^3 \ + \ (T_{y2}^2 \ + \ T_{yz} T_{y1}) s^2 \ + \ (T_{y1} + T_{yz}) s + (1+k_y) \ = \ 0$$

For harmonic solutions to exist at the stability limit (σ = 0), substituting $s = j\omega$ an equating the real and imaginary part to zero gives,

$$\omega^3 T_{yz} \ T_{y2}^2 \ + \ (T_{y1} \ + \ T_{yz}) \ \omega \ = \ 0 \qquad\qquad (22a)$$

$$-(T_{y2}^2 \ + \ T_{yz} \ T_{y1}) \ \omega^2 \ + \ (1+k_y) \ = \ 0 \qquad\qquad (22b)$$

Therefore,

$$\omega^2 \ = \ \frac{1 \ + \ k_y}{T_{y2}^2 \ + \ T_{yz} \ T_{y1}} \ = \ \frac{T_{y1} \ + \ T_{yz}}{T_{yz} \ T_{y2}^2} \qquad (23)$$

from (23)

$$k_y \ \approx \ d_{ty} \ (D \ + \ \frac{1}{D}) \qquad\qquad (24)$$

where $d_{ty} = \dfrac{b_{ty}\omega}{c_{ty}}$ and $D = \omega \; \omega \; T_{yz}$

and

$$\omega = \sqrt{\frac{1 + k_y}{T_{y2}^2 + T_{yz}T_{y1}}} \qquad (25)$$

In order to convert equation (14) into the form of equation (20), the energy equivalence technique has been used [9].

Writing,

$$T_{ky12}\dot{y} + T_{kz12}\dot{z} = (T_{ky12} + \eta_{yz} T_{kz12})\dot{y}$$

$$= T_1 \dot{y} \qquad (26)$$

where T_1 is the relative damping and η_{yz} is the coefficient that takes into account the vibration energy introduced by $T_{kz12}\dot{z}$ into the system. This helps in formally excluding the effect of z direction on y direction. However, as the total energy in a cycle remains constant, the stability conditions remain the same.

Now, $\eta_{yz} T_{kz12}\dot{z}$ and $T_{ky12}\dot{y}$ are essentially damping forces moving the total mass m_t with velocity $\dot{y}$. Therefore, from the consideration that the total energy remains constant

$$\eta_{yz} \int_T \left[\frac{T_{kz12}\,\Delta\dot{y}}{y_o} \cdot \frac{\Delta\dot{y}}{y_o} \right] dt = \int_T \left[\frac{T_{kz12}\,\Delta\dot{z}\cdot\Delta\dot{y}}{z_o y_o} \right] dt$$

$$\therefore \eta_{yz} = \frac{\int_T [\dot{\Delta z} \; \dot{\Delta y}]dt}{z_o \int_T [\dot{\Delta y}^2]dt} \qquad (27)$$

Now, $c_{ty}y_o = \eta \; c_{tz1}z_o$

$$\therefore \frac{y_o}{z_o} = \frac{\eta \; c_{tz}}{c_{ty}}$$

$$\therefore \eta_{yz} = \frac{\eta c_{tz1}}{c_{ty}} \cdot \frac{\int_T [\dot{\Delta y}\dot{\Delta z}]dt}{\int_T [\dot{\Delta y}^2]dt} \qquad (28)$$

Taking $\Delta y = A_y \; Sin(\omega t)$

and $\Delta z = A_z \; Sin(\omega t - \phi_{yz})$

where,

$\phi_{yz} = $ lag of Δz w.r.t. Δy,

$$\eta_{yz} = \frac{\eta \; c_{tz1}}{c_{ty}} \cdot \frac{A_z}{A_y} [\cos \phi_{yz} + (0.5/\pi)Sin \phi_{yz}]$$

$$= \frac{\eta \; c_{tz}}{c_{ty}} \cdot \frac{A_z}{A_y} \cdot [X1] \qquad (29)$$

where,

$$X1 = Cos \phi_{yz} + (0.5/\pi) Sin \phi_{yz} \qquad (30)$$

Another method of excluding the effect of $\Delta \dot{z}$ is by converting it it in terms of $\Delta \dot{y}$ as shown below:

$$\Delta \dot{z} = \omega A_z \cos(\omega t - \phi_{yz})$$

$$= \omega A_z \cos\omega t \, \cos\phi_{yz} + \omega A_z \sin\omega t \, \sin\phi_{yz}$$

Now, $A_z = \dfrac{A_{Fz}}{c_{tz}} V_1(g)$ where $V_1(g) = \dfrac{1}{\sqrt{(1-g^2)^2 + (d_{tz1}g)^2}}$

(30a)

$$= \dfrac{K \, A_y \, V_1(g)}{c_{tz} \sqrt{1 + \omega^2 T_z^2}}$$

(30b)

$$= K_1 \, A_y$$

where, $K_1 = \dfrac{K \, V_1(g)}{c_{tz1} \sqrt{1 + \omega^2 T_z^2}}$

$\therefore \quad \Delta \dot{z} = \omega K_1 A_y \cos(\omega t - \phi_{yz})$

$$= \omega K_1 \, A_y \cos\omega t \, \cos\phi_{yz} + \omega \, K_1 A_y \sin\omega t \, \sin\phi_{yz}$$

$$= K_1 \cos\phi_{yz} \Delta \dot{y} + K_1 \sin\phi_{yz} \, \tan\omega t \, \Delta \dot{y}$$

$$\left[\begin{array}{l} \text{Since } \Delta y = A_y \, \text{Sin}\omega t \\[8pt] \therefore \dot{\Delta y} = \omega A \, \text{Cos}\omega t \end{array} \right]$$

$$= K_1 \, [\text{Cos}\phi_{yz} + \tan\omega t \, \text{Sin}\phi_{yz}] \dot{\Delta y}$$

$$\dot{\Delta z} = K_2 \, \dot{\Delta y} \qquad\qquad (30c)$$

where, $K_3 = [\text{Cos}\phi_{yz} + \tan\omega t \, \text{Sin}\dot{\phi}_{yz}]$ $\qquad\qquad (30d)$

and $\quad K_2 = K_1 K_3$ $\qquad\qquad\qquad (30e)$

$$\frac{\dot{\Delta z}}{z_o} = K_2 \cdot \frac{\eta \, c_{tz}}{c_{ty}} \, \dot{y}$$

$$= K_1 K_3 \cdot \frac{\eta c_{tz1}}{c_{ty1}} \, \dot{y}$$

$$= \frac{K \cdot V_1(g)}{\sqrt{1 + \omega^2 T_t^2}} \quad x \quad K_3 \cdot \frac{\eta \, c_{tz1}}{c_{ty}} \dot{y}$$

$$= \frac{k_y \, K_3 \, V_1(g)}{\sqrt{1 + \omega^2 T_z^2}} \, \dot{y} \qquad\qquad (30.f)$$

Similarly, taking

$$T_y \dot{F}_y + T_z \dot{F}_z = (T_y + \eta_{Fyz} \, T_z) \dot{F}_y$$

where η_{Fyz} compensates for the energy absorbed by $v_s T_z \dot{F}_z$ from the y directions

$$\therefore \eta_{Fyz} \int_T |T_z F_y \ddot{y}| \, dt = \int |T_z \dot{F}_z \dot{z}| \, dt$$

$$\therefore \eta_{Fyz} = \frac{\int_T |\dot{F}_z \dot{z}| \, dt}{\int_T |\dot{F}_y \dot{y}| dt}$$

$$= \frac{\eta \, A_{Fz}}{A_{Fy}} = Cos\phi_1 / Cos\phi_3 \qquad (31)$$

where, ϕ_1 = lag of ΔF_z w.r.t. Δy

ϕ_3 = lag of ΔF_y w.r.t. Δy

$= \phi_1 + \phi_2$

ϕ_2 = lag of ΔF_y w.r.t. ΔF_z.

thus equation (14) may be written as,

$$T_{yz}^* \, \dot{T}_y + F_y + k_y \, y - T_{ky}^* \, \dot{y} = 0 \qquad (32)$$

where,

$$T_{yz}^* = T_y + \eta_{Fyz} \, T_z \qquad (33a)$$

and $\quad T_{ky}^* = T_{ky12} + \eta_{yz} \, T_{kz12} \qquad (33b)$

Equation (32) after substitution of (16b) and Laplace transform becomes a third order system as shown:

$$T_{yz}^* \, T_{y2}^2 \, s^3 + (T_{yz}^* \, T_{y1} + T_{y2}^2) \, s^2$$

$$+ \, [(T_{y1} + T_{yz}^*) - T_{ky}^*] \, s + (1 + k_y) = 0.$$

Substituting $s = j\omega$ at the stability limit,

$$k_y = d_{ty} \left(D + \frac{1}{D} \right) - \frac{\omega \, T_{ky}^*}{D} \, (1 + d_{ty} D)$$

where $D = \omega T_{yz}$ and

$$\omega = \sqrt{\frac{1 + k_y}{T_{y2}^2 + T_{yz}^* \, T_{y1}}}$$

neglecting $d_{ty} D$ which is small

$$k_y = d_{ty} \left(D + \frac{1}{D} \right) - \frac{\omega \, T_{ky}^*}{D} \qquad (34)$$

Thus, in a two degree of freedom system k_y gets modified to the form as shown in equation (34) from that in equation (24).

While calculating k_y to find out the stability limit in the beginning it is assumed that $\omega = \omega_y$ and subsequent calculations are carried out. The value $\omega = \omega_y$ is selected since chatter occurs at a frequency close to the natural frequency of the system (see first chapter).

Determination of A_y, A_{Fz}, A_{Fy}, A_z:

     At any time t, the instantaneous chip thickness

$$s(t) = s_o + y(t) - \mu y(t - T)$$

$$\therefore \quad \Delta s(t) = s(t) - s_o$$

$$= y(t) - \mu y(t - T)$$

where $y(t-T)$ is the chip thickness one revolution earlier

μ = overlap factor

       = 0 for the first cut ⎤ for regenerative

       = 0-1 for subsequent cut ⎟ chatter

       = 0 for primary chatter ⎦

Assuming that the chip thickness variation differs little from harmonic

$$\Delta s(t) = y(t) - \mu y(t - T)$$

$$= A_y \sin \omega t - \mu A_y \sin(\omega t - \theta) \qquad (35)$$

where, θ = phase angle between two successive cuts

$$= \frac{2\pi \omega}{\omega}$$

$$= \frac{60 \cdot \omega}{N} \quad \begin{bmatrix} W = \text{chatter frequency} \\ N = \text{r.p.m. of work} \end{bmatrix}$$

from (35)

$$s(t) = A_y \sin\omega t - \mu A_y [\sin\omega t \cos\theta - \cos\omega t \sin\theta]$$

$$= A_y \sin\omega t [1 - \mu\cos\theta] + \mu A_y \cos\omega t \sin\theta$$

$$= m A_y \sin(\omega t + \theta_1) \qquad\qquad (36)$$

where, $\quad m = \{ (1 - \mu\cos\theta)^2 + (\mu\sin\theta)^2 \}$

$$\theta_1 = \tan^{-1} \left(\frac{\mu\sin\theta}{1 - \mu\cos\theta} \right).$$

Thus, the effect of the overlap factor is to increase the amplitude of vibrations and introduce a phase lead of the resultant variation w.r.t. y(t). Obviously, if there is now a lag of the cutting forces much more energy will be available to cause instability. Since the components of the cutting force lag behind the chip thickness variations; therefore,

$$\Delta F_z = A_{Fz} \sin(\omega t - \varphi_1) \qquad\qquad (37a)$$

$$\Delta F_y = A_{Fy} \sin(\omega t - \varphi_2) \qquad\qquad (37b)$$

where φ_1 and φ_2 are as defined previously (see eq. 31). Under linearised conditions equation (13b) can be written as

$$\Delta F_y + T_y \Delta F_y = \eta \Delta F_z \qquad\qquad (38)$$

Substituting (37) in (38) and equating the coefficient of $\sin\omega t$ and $\cos\omega t$ to zero gives,

$$\left|\frac{A_{Fy}}{A_{Fz}}\right| = \frac{n}{\sqrt{1 + \omega^2 T_y^2}} \qquad (39a)$$

and $\quad \gamma_2 = \tan^{-1}(\omega T_y) \qquad (39b)$

Similarly, since for small vibrations, $\omega z \ll v_s$; (13a) can be written under such conditions

$$T_z \, \omega \dot{F}_z + \omega F_z = -K \omega y \qquad (40)$$

Proceeding similarly,

$$\left|\frac{A_{Fz}}{A_y}\right| = \frac{K}{\sqrt{1 + \omega^2 T_z^2}} \qquad (41a)$$

$$\gamma_1 = \tan^{-1}(\omega T_z) \qquad (41b)$$

from the figure (2.3)

$$\omega y(t) = A_y \sin \omega t$$
$$l_z(t) = v_s t - A_z \sin(\omega t - \phi_{yz})$$
$$= v_s t + A_z \sin(\omega t + \theta_z)$$

where, $\quad \theta_z = \pi - \phi_{yz}.$

From the figure it is evident that the condition $\min(\omega y/\dot{l}_z)$ $\gg \tan\gamma_c$ helps find out the permissible limit cycle amplitude to prevent rubbing along the clearance face,

$$\left|\frac{A_{Fy}}{A_{Fz}}\right| = \frac{\eta}{\sqrt{1 + \omega^2 T_y^2}} \tag{39a}$$

and $\quad \phi_2 = \tan^{-1}(\omega T_y) \tag{39b}$

Similarly, since for small vibrations, $\dot{\Delta z} << v_s$; (13a) can be written under such conditions

$$T_z \Delta \dot{F}_z + \Delta F_z = -K \Delta y \tag{40}$$

Proceeding similarly,

$$\left|\frac{A_{Fz}}{A_y}\right| = \frac{K}{\sqrt{1 + \omega^2 T_z^2}} \tag{41a}$$

$$\phi_1 = \tan^{-1}(\omega T_z) \tag{41b}$$

from the figure (2.3)

$$\Delta y(t) = A_y \sin \omega t$$

$$l_z(t) = v_s t - A_z \sin(\omega t - \phi_{yz})$$

$$= v_s t + A_z \sin(\omega t + \phi_z)$$

where, $\quad \phi_z = \Pi - \phi_{yz}.$

From the figure it is evident that the condition $\min(\dot{\Delta y}/\dot{l}_z)$ $\geq - \tan \phi_c$ helps find out the permissible limit cycle amplitude to prevent rubbing along the clearance face,

$$\frac{\Delta \dot{y}}{\dot{1}_z(t)} = -\tan \alpha_c \quad \text{gives}$$

$$\frac{\omega A_y \cos \omega t}{V_s + A_z \cos (\omega t + \theta_z)} = -\tan \alpha_c$$

if

$$\left. \begin{array}{rcl} F_1 & = & v_s \tan \alpha_c \\[2mm] F_2 & = & A_z \tan \alpha_c \end{array} \right] \tag{42}$$

then from development and mathematical manipulations it can be shown

$$A_y = \frac{V_s \tan \alpha_c}{\omega} \cdot \frac{1}{\sqrt{1+(F_2/A_y)^2 + 2(F_2/A_y)\cos \theta_z}}$$

and $\sin(\omega t) = \dfrac{V_s A_z \tan^2 \alpha_c}{\omega A_y^2 [1+ (F_2/A_y)^2 + 2(F_2/A_y)\cos \theta_z]}$ \hfill (42.b)

To find T_{ky12} and T_{kz12} :

$$\text{Now, } T_{ky12} = \frac{\varepsilon_1 (\omega \Delta F_z - \Delta F_y)}{v_s c_{ty}} \quad (\text{See Eqn.14})$$

$$= \frac{\varepsilon_1 \omega \Delta F_z}{v_s c_{ty}} - \frac{\varepsilon_1 \Delta F_y}{v_s c_{ty}}$$

$$= T_{ky1} - T_{ky2}.$$

Now, $T_{ky12} \cdot \Delta \dot{y}$ represents some amount of vibrational energy and it will introduce positive or negative damping into the system depending upon the phases interaction.

Writing $\Delta F_z \Delta \dot{y} = T_{ky1} \Delta \dot{y}$ on has from the equivalence that the total energay remains constant;

$$T_{ky1} = \frac{\int_T \Delta F_z \Delta \dot{y} \, dt}{\int \Delta \dot{y} \, dt}$$

with $\Delta F_z = A_{Fz} \sin \omega t$ and $\Delta y = A_y \sin(\omega t + \dot{\phi}_1)$

$$T_{ky1} = \frac{\omega \int_0^{2\pi/\omega} A_{Fz} \sin \omega t \, \cos(\omega t + z_1) \, dt}{4\omega \int_0^{\pi/2\omega} \cos(\omega t + \dot{\phi}_1) \, dt}$$

$$= - \frac{\pi \sin z_1}{4(\cos \phi_1 - \sin \phi_1)} \cdot A_{Fz}$$

$$= - \frac{\pi \sin \dot{\phi}_1}{4(\cos \dot{\phi}_1 - \sin \dot{\phi}_1)} \cdot \frac{K A_y}{\sqrt{1 + \omega^2 T_z^2}} \qquad (43a)$$

similarly,

$$T_{ky2} = \frac{\int_0^{2\pi/\omega} \Delta F_y \Delta \dot{y} \, dt}{4 \int_0^{\pi/2\omega} \Delta \dot{y} \, dt}$$

$$= - \frac{\Pi \sin \varphi_3}{4(\cos \varphi_1 - \sin \varphi_1)} \frac{\eta K A_y}{\sqrt{(1+\omega^2 T_y^2)(1+\omega^2 T_z^2)}} \qquad (43b)$$

thus,

$$T_{ky12} = \frac{\Pi \varphi_1 \eta K A_y}{4(\cos \varphi_1 - \sin \varphi_1)} \left[\frac{\sin \varphi_3}{\sqrt{(1+\omega^2 T_y^2)(1+\omega^2 T_z^2)}} - \frac{\sin \varphi_1}{\sqrt{1+\omega^2 + T_z^2}} \right] \cdot \frac{1}{v_s c_{ty}}$$

$$(43c)$$

and, $$T_{kz12} = \frac{\displaystyle\int_0^{2\Pi/\omega} \Delta F_y \, \Delta \dot{z} \, dt}{4 \displaystyle\int_0^{\Pi/2\omega} \Delta \dot{z} \, dt}$$

$$= \frac{4\Pi \sin(\varphi_{Fz} - \varphi_2) - \cos(\varphi_{Fz} + \varphi_2)}{16(\sin \varphi_{Fz} + \cos \varphi_{Fz})} \frac{KA_y}{\sqrt{(1+\omega^2 T_z^2)(1+\omega^2 T_y^2)}}$$

$$(43d)$$

where,

$$\varphi_{Fz} = \text{lag of } \Delta z \text{ behind } \Delta F_z$$

$$= \frac{d_{tz1} g}{1 - g^2} \qquad (44)$$

and

d_{tz1} = damping of the main system in the tangential directio

g = ω/ω_{z1}

Now, one can see that T_{ky12} always introduces negative damping into the system and, hence, lowers the system stability. On the other hand T_{kz12} can introduce negative or positive damping into the system depending upon the relationship between the various phases. Thus, while considering methods to improve the stability, one way can be trying to modify the system parameters in such a way so that either (a) T_{kz12} becomes negative or (b) A_y is reduced.

2.2 Stability of the System with the Absorber

A dynamic absorber consisting of a spring-mass damper when attached to the main system shifts the resonant peaks of the main system away from the forcing frequency. Due to the presence of the absorber, the amplitude of vibration of the main mass is reduced by transferring the vibrational energy across the damper. The energy is expended usually by causing the small absorber mass to undergo large amplitudes. Thus although the main system has its amplitude reduced but the overall amplitude of vibrations is increased.

In the present system the absorber is attached to the tangential direction because of the following reasons:

a) Since the present absorber is a passive type and, hence, tuning being critical, any mismatch will produce adverse effects upon depth of cut;

b) Since vibrations in the tangential directions with a sharp tool do not initiate chatter but affect stability limits [4], therefore by improving the compliance in this direction one can affect the overall stability limit;

c) modifying the phase relationship and at the same time trying to increase the overall compliance will introduce greater positive damping into the system since as stated previously [14] stability limit can be improved even by increasing compliance as the additional degrees of freedom introduced increases the effective damping in the system through mutual phase interaction;

d) Since compliance in the tangential direction is more than that in the thrust direction [18,19,20] as a result of reduced stiffness due to tool overhang, any improvement in the compliance of the tangential direction will help improving stability.

In the new system with the absorber, it is assumed that the amplitude and phase relationship still depend upon the y direction and are completely independent of the z direction and the vibrations in the tanagential direction still depend upon ΔF_z only.
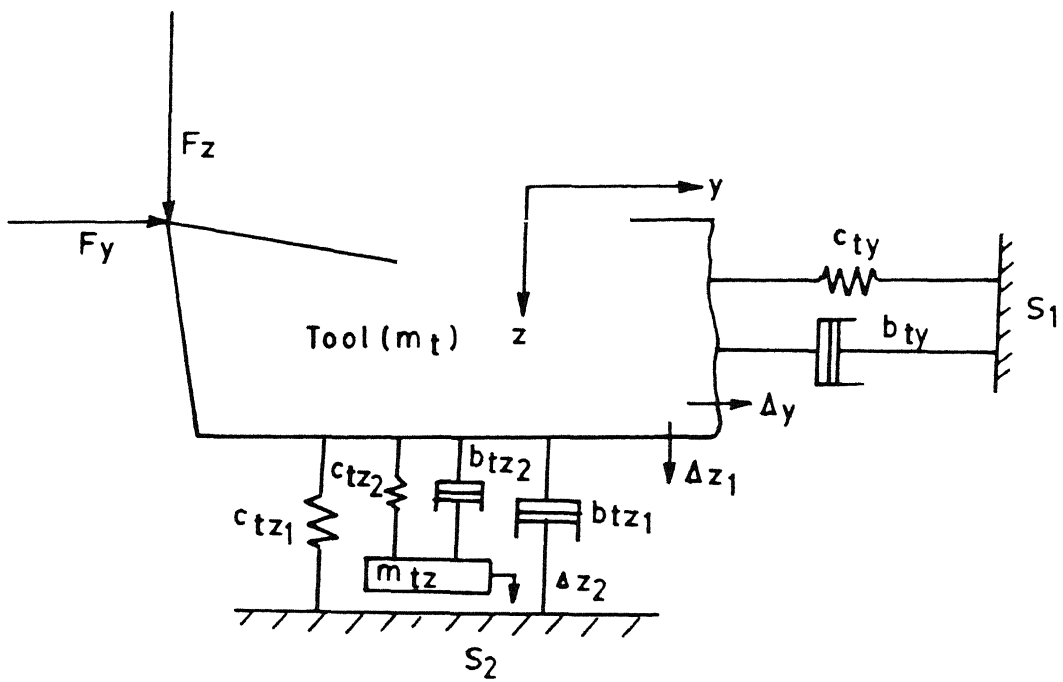
Fig. 2·5   Vibration model of cutting process with absorber

Equations of Motion (with absorber) in the tangential direction

The governing equationsof motion are (Fig. 2.5)

$$m_t \, \Delta\ddot{z} \; + \; b_{tz1} \, \Delta\dot{z}_1 \; - \; c_{tz1} \Delta z$$

$$+ \; c_{tz2} \, (\Delta z_1 - \Delta z_2) + b_{tz2}(\Delta\dot{z}_1 - \Delta\dot{z}_2) \; = \; \Delta F_z \qquad (45a)$$

$$m_{tz} \Delta\ddot{z}_2 \; - \; c_{tz2}(\Delta z_1 - \Delta z_2) \; - \; b_{tz2}(\Delta\dot{z}_1 - \Delta\dot{z}_2) \; = \; 0 \qquad (45b)$$

Substituting $\quad \Delta F_z \quad = \quad A_{Fz} \; e^{i\omega t}$

$$\Delta z_1 \quad = \quad A_{z1} \; e^{jwt}$$

$$\Delta z_2 \quad = \quad A_{z2} \; e^{j\omega t}$$

one obtains, through mathematical manipulation

$$\frac{A_{z1}}{(A_{Fz}/c_{tz1})} \; = \; \left[\frac{(g^2 - f^2)^2 + (d_{tz2} \, gf)^2}{(\mu_1 f^2 g^2 - (g^2 - 1)(g^2 - f^2))^2 \; + \; (d_{tz2} \, gf)^2 (\mu_1 p \, (g^2 - f^2) + (g^2 - 1) + \mu_1 g^2)^2} \right]^{1/2}$$

$$(46a)$$

$$A_{z2}/A_{z1} \; = \; \sqrt{\frac{f^4 + (d_{tz2} \, gf)^2}{(g^2 f^2)^2 + (d_{tz2} gf)^2}} \qquad (46b)$$

where $\quad g \; = \; \omega/\omega_{z1} \quad = \quad \dfrac{\text{forcing frequency}}{\text{natural frequency of main mass}}$

$$f = w_{z2}/w_{z1} = \frac{\text{natural frequency of absorber}}{\text{natural frequency of main mass}}$$

$$p = \frac{b_{tz1}}{b_{tz2}} = \frac{\text{damping in main mass}}{\text{damping in absorber}}$$

$$\mu_1 = m_{tz}/m_t = \text{mass ratio.}$$

In the presence of the absorber the lag of Δz_1 w.r.t. ΔF_z becomes

$$\phi_{Fz1} = \tan^{-1}\left[\frac{(gfd_{tz2})((f^2-g^2)[\mu_1 p(g^2-f^2)+(1-g^2)-\mu_1 g^2]}{-[(g^2-1)(g^2-f^2)-\mu_1 f^2 g^2]))}{(f^2-g^2)((g^2-1)(g^2-f^2)-\mu_1 f^2 g^2)+(d_{tz2}gf)^2} \right] \over \{\mu_1 p(g^2-f^2)+(1-g^2)-\mu_1 g^2\}$$

$$(47a)$$

and the lag of Δz_2 w.r.t. Δz_1

$$\phi_{z12} = \tan^{-1}\left[\frac{g^3 f\, d_{tz2}}{f^2(f^2-g^2)+(d_{tz2}gf)^2}\right] \qquad (47b)$$

The overall vibrational amplitude in the z direction,

$$A_{ze} = \sqrt{A_{z1}^2 + A_{z2}^2 + 2A_{z1}A_{z2}\cos\phi_{z12}} \qquad (47c)$$

In order to find out the values of g so that $A_{z1}/(A_{Fz}/c_{tz1})$ becomes independent of the damping in the absorber,

$$\left[\frac{g^2 - f^2}{\mu_1 f^2 g^2 - (g^2-1)(g^2-f^2)}\right]^2 = \frac{1}{[\mu_1 p(g^2-f^2) + (g^2-1) + \mu g^2]^2}$$

working with the positive root,

$$\mu_1 f^2 g^2 - (g^2-1)(g^2-f^2) = \mu_1 P(g^2-f^2) + (g^2-1)(g^2-f^2) + \mu_1 g^2(g^2-f^2)$$

or,

$$g^4(2 + \mu_1 P + \mu_1) - 2g^2(\mu_1 Pf^2 + \mu_1 f^2 + f^2 + 1) + (2f^2 + \mu_1 Pf^4) = 0$$

$$\therefore \quad g_1^2 + g_2^2 = \frac{2(\mu_1 Pf^2 + \mu f^2 + f^2 + 1)}{2 + \mu_1 P + \mu_1} \tag{48}$$

Now, since, $A_{z1}/(A_{Fz}/c_{tz1})$ is independent of damping therefore one can choose any value for damping in the absorber. Choosing $b_{tz2} = \infty$ to get the simplest form of $A_{z1}/(A_{Fz}/c_{tz1})$ one has

$$A_{Fz1}/(A_{Fz}/c_{tz1}) = \frac{1}{1 - g_2^2(1 + \mu_1)} \tag{49}$$

taking the negative root;

$$\frac{1}{1 - g_1^2(1 + \mu_1)} = -\frac{1}{1 - g^2(1 + \mu_1)}$$

$$\therefore \quad g_1^2 + g_2^2 = \frac{2}{1 + \mu_1} \tag{50}$$

fro eq. (48) and eq. (50),

$$f = \left[\frac{1 + \mu_1 P}{(1 + \mu_1)(\mu_1 P + \mu_1 + 1)} \right]^{1/2} \tag{51}$$

from eq. (51) two interesting cases may be observed; for $P = \infty$ i.e. for the case when the damping in the absorber is absent

$$f = [1/(1 + \mu_1)]^{1/2} \tag{52a}$$

and in the case when the main system has no damping, i.e, $p = 0$

$$\therefore \quad f = 1/(1 + \mu_1) \tag{52b}$$

from (47.a) and (47.b),

$$\varphi_{Fz2} = \varphi_{Fz1} = \varphi_{z12} \tag{47.d}$$

and $\quad \varphi_{Fz12} = \tan^{-1} \left[\frac{A_{z1} \sin\varphi_{Fz1} + A_{z2} \sin\varphi_{Fz2}}{A_{z1} \cos\varphi_{Fz1} + A_{z2} \sin\varphi_{Fz2}} \right]$ $\tag{47.3}$

Considering the case when the damping in the absorber is absent, eq. (47a) is moidified as,

$$A_{z1}/(A_{Fz}/c_{tz1}) = \frac{g^2 - f^2}{[(\mu_1 f^2 g^2 - (g^2 - f^2)(g^2 - 1))^2 + (d_{tz1} g)^2 (g^2 - f^2)^2]^{1/2}}$$

$$\tag{51a}$$

$$A_{z2}/A_{z1} = \frac{f^2}{g^2 - f^2} \tag{53.b}$$

In this case, the lag of $\triangle z_1$ w.r.t. $\triangle F_z$ can be expressed as,

$$\phi_{Fz1} = \tan^{-1}\left[\frac{(d_{tz1}\,g)\,(g^2 - f^2)}{\mu_1\,f^2 g^2 - (g^2 - f^2)(g^2 - 1)}\right] \tag{53.c}$$

$$\phi_{z12} = 0 \tag{53.d}$$

so that $\phi_{Fz2} = \phi_{Fz1} = \phi_{Fz12}$ \hfill (53.e)

and the overall amplitude $A_{ze} = A_{z1} + A_{z2}$ \hfill (53.f)

Substituting A_{ze} and ϕ_{Fz12} from equations (53.e) and (53.f) or equations (47.c) and (47.e) as the case may be depending on the conditionof absorber, helps to find out the new value of k_y and thereby the new stability limit.

In the presence of the absorber,

$$V(g) = \sqrt{V_1(g)^2 + V_2(g)^2 + 2V_1(g)\,V_2(g)\,\cos\phi_{z12}}$$

where,

$$V_1(g) = A_{Fz1}/(A_{Fz}/c_{tz1})$$

and $\qquad V_2(g) = A_{Fz2}/(A_{Fz}/c_{tz1})$

and $\qquad K_3 = \cos\phi_{yz} + \tan\omega t\,\sin\phi_{yz}$

where,

$$\phi_{yz} = \phi_{Fz12} + \phi_1,$$

2.3 Theoretical Results and Analysis:

Table 1 compares tehe improvement under cases when the absorber is a pure spring with no damping (A) and when the main system is assumed to have no damping (B) under a particular cutting condition. It can be seen (Fig.2.6) that with increasing mass ratio the improvement achieved by incorporating the absorber falls down. However, the decrease in (A) is less than that in (B).

With increasing cutting speed (Table 2, fig. (2.7)) for a given tuning condition, these exists a minimum value of k_v in both cases, although the decrease in (D) is less than that in (C). At high cutting speeds, the improvement in (C) shows improved characteristics.

Table (3) compares the variation of k_1 as a function of the ratio of timing under optimum conditions for a given cutting condition. From Table (3) and Fig. (2.8) it is evident that (E) gives better performance as compared to that of (F).

Thus from the theoretical analysis it can be concluded that,

(a) if the desire is to operate at a particular cutting speed at optimum tuning conditions, using a spring as absorber with as little damping as possible will provide better result. (Fig.(2.8)). However the improvement will fall down as the mass ratio is increased (Fig. (2.6)).

(b) at low cutting speeds, a pure spring (D) will provide better results but at higher speed ranges better results can be

expected from (C). Thus if the main system has very low damping, a damped absorber will provide better results.

Under practical conditions, it is very difficult to come across an absorber that has no damping at all. Thus for practical conditions, the ratio "P" has to be determined experimentally and the proper tuning has to be set accordingly. Obviously, the improvement in that case will lie in between the two extreme conditions analysed here.

Table 1

Variation of $k_{\mu 1}$ under optimum tuning conditions as a function of mass ratio (μ)

Mass Ratio	Tuning Ratio		$k_1 = \dfrac{\text{dyn.stiffness with absorber}}{\text{dyn.stiffness without absorber}}$	
μ	(A) $f=1/\sqrt{1+\mu_1}$	(B) $f=1/(1+\mu_1)$	(A)	(B)
0.10	0.95	0.91	1.53	1.23
0.15	0.9325	0.8695	1.3	1.05
0.20	0.9128	0.834	1.23	1.07
0.25	0.8944	0.80	1.215	1.07
0.30	0.877	0.7692	1.20	1.077
0.35	0.86	0.740	1.1846	1.08
0.40	0.845	0.714	1.177	1.085

B EXPERIMENTAL RESULT (SET-2)

tangential vibrations
x 10^{-3} cm.

k$_i$

IMPROVEMENT COEFFICIENT k$_i'$

GAP LENGTH (mm)

2.00 1.94 1.87 1.81 1.74 1.68 1.61 1.55 1.48 1.42 1.35 1.29 1.22 1.16 1.09 1.02 0.96 0.90 0.83 0.76 0.70

18.00 24.20 30.40 36.60 42.80 49.00 55.20 61.40 67.60 73.80 80.00
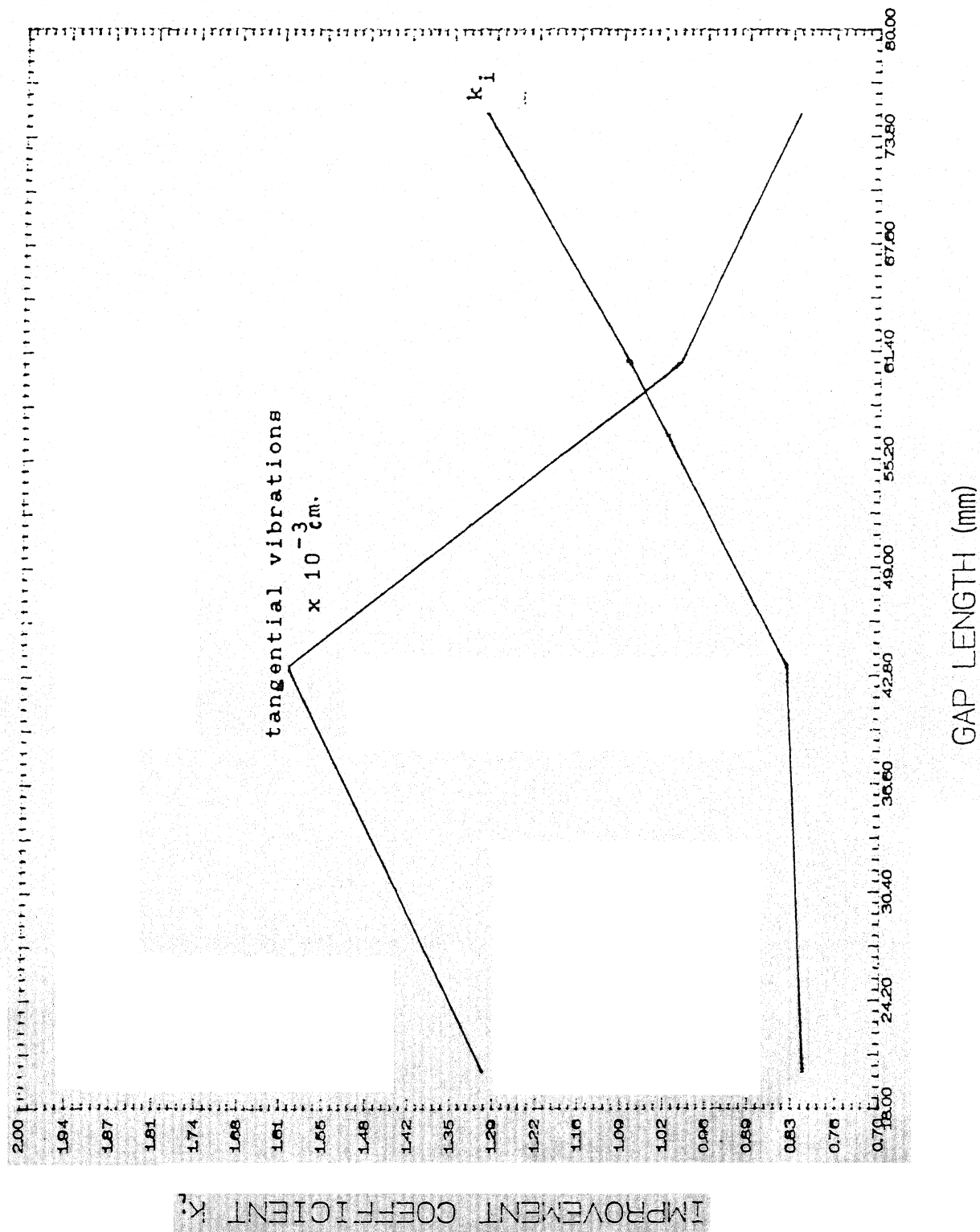
Table 2

Variation of k_v under optimum tuning conditions as a function of

cutting speed

Cutting speed	k_v	=	dyn. stiffness with absorber
			dyn. stiffness without absorber

(m/s)	(C) $f = 1/(1 + \mu_1)$	(D) $f = 1/\sqrt{(1 + \mu_1)}$
1.0	1.165	1.165
1.5	1.10	1.175
2.0	0.98	1.08
2.5	1.23	1.24
3.0	1.24	1.18
3.5	1.34	1.23
4.0	1.45	1.407

Table 3

Variation of k_1 as a function of the ratio of

tuning under optimum condition

$$K_1 = \frac{\text{dyn. stiffness with absorber}}{\text{dyn. stiffness without absorber}}$$

n	(E) $f = n/(1+\omega_1)$	n_1 (F)	$f_1 = n/\sqrt{1+\omega_1}$
0.4	1.19	0.4	0.623
0.6	1.17	0.6	0.29
0.8	1.104	0.8	1.36
1.0	1.248	1.0	1.538
1.2	1.28	1.1	1.292
1.4	1.215	1.2	1.19
1.6	1.223	1.4	1.15
1.8	1.216	1.6	1.138
2.0	1.20	1.8	1.13
		2.0	1.115

Fig 2.6 VARIATION OF k_i VS μ_i

$f = 1/\sqrt{(1 + \mu_1)}$

$f = 1/(1 + \mu_1)$

MASS RATIO (μ_i)

IMPROVEMENT COEFFICIENT (k_i)

Fig 2.7 VARIATION OF k_i VS v

$f = 1/(1 + \mu_1')$

$f = 1/\sqrt{(1 + \mu_1')}$

IMPROVEMENT COEFFICIENT k_i

CUTTING SPEED v (m/sec)
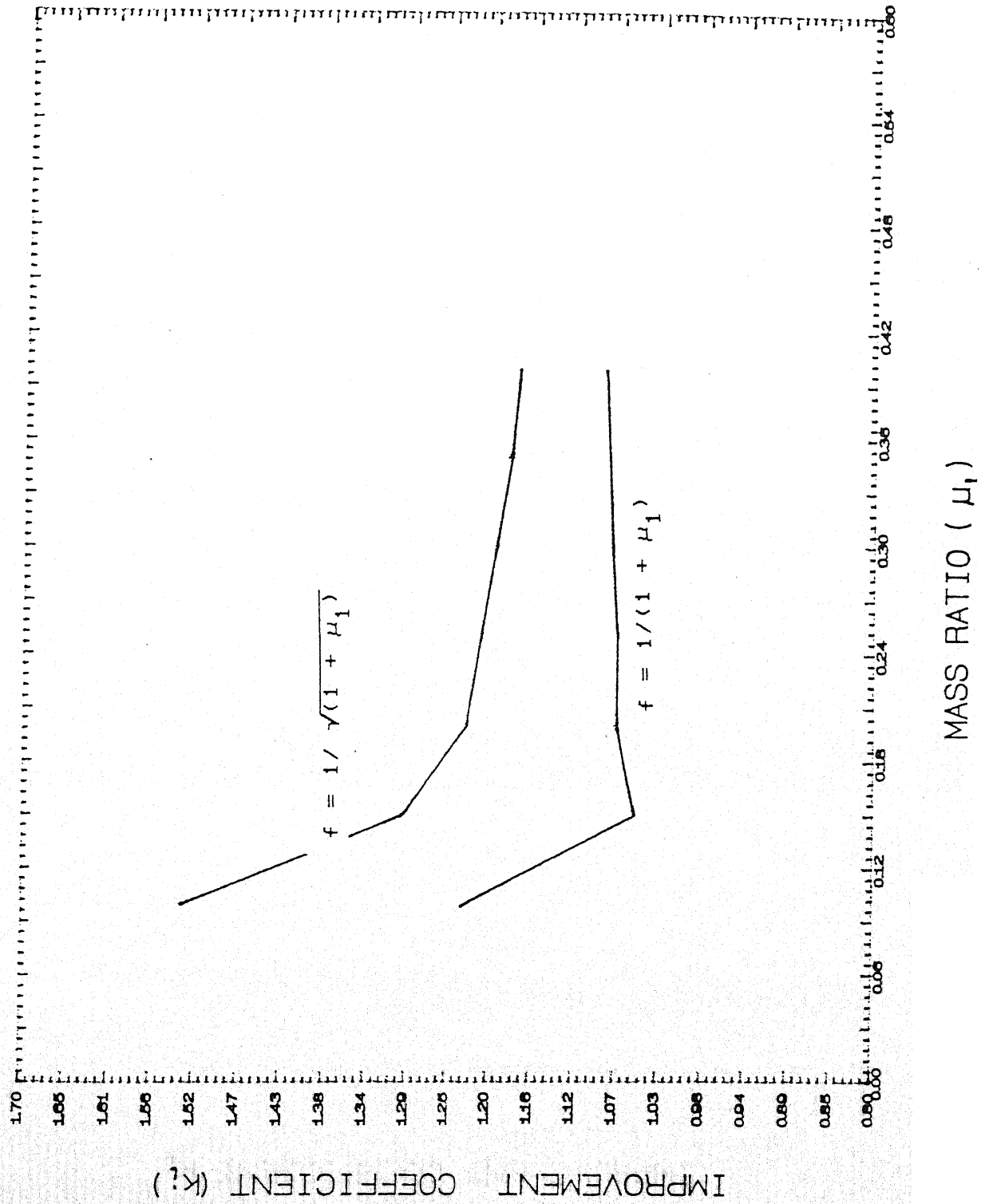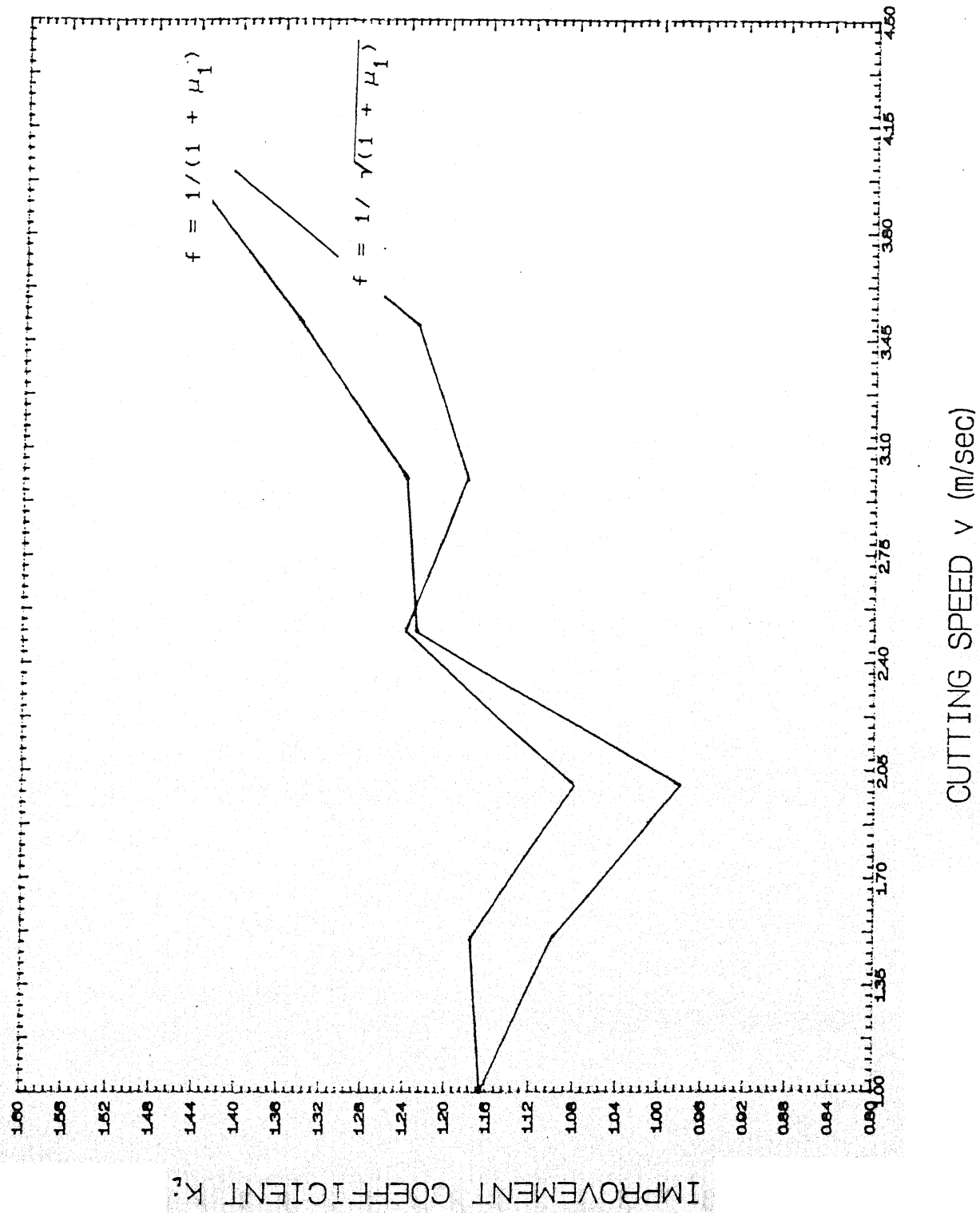
Fig 2.8 VARIATION OF k_i VS f

$$f = 1/(1 + \mu_1)$$

$$f = 1/\sqrt{(1 + \mu_1)}$$

IMPROVEMENT COEFFICIENT (k_i)

1.70
1.62
1.54
1.46
1.38
1.30
1.22
1.14
1.06
0.98
0.90
0.82
0.74
0.66
0.58
0.50
0.42
0.34
0.26
0.18
0.10

0.30
0.48
0.66
0.84
1.02
1.20
1.38
1.56
1.74
1.92
2.10

TUNING RATIO (f)

B EXPERIMENTAL RESULT (SET-2)

tangential vibrations
x 10^{-3} cm.

k_i

IMPROVEMENT COEFFICIENT k_i

2.00
1.94
1.87
1.81
1.74
1.68
1.61
1.55
1.48
1.42
1.35
1.29
1.22
1.16
1.09
1.02
0.96
0.89
0.83
0.76
0.70

18.00
24.20
30.40
36.60
42.80
49.00
55.20
61.40
67.60
73.80
80.00

GAP LENGTH (mm)
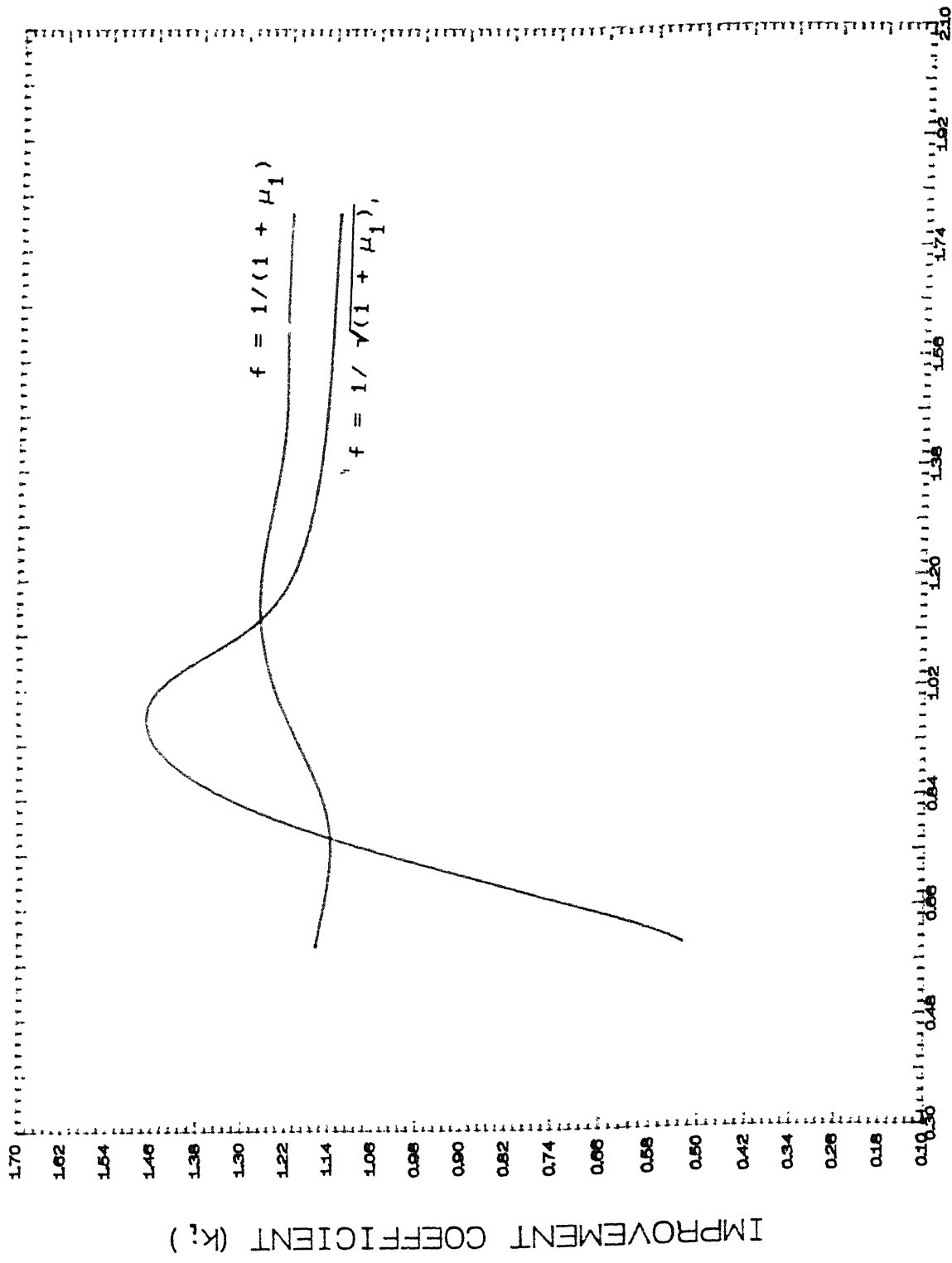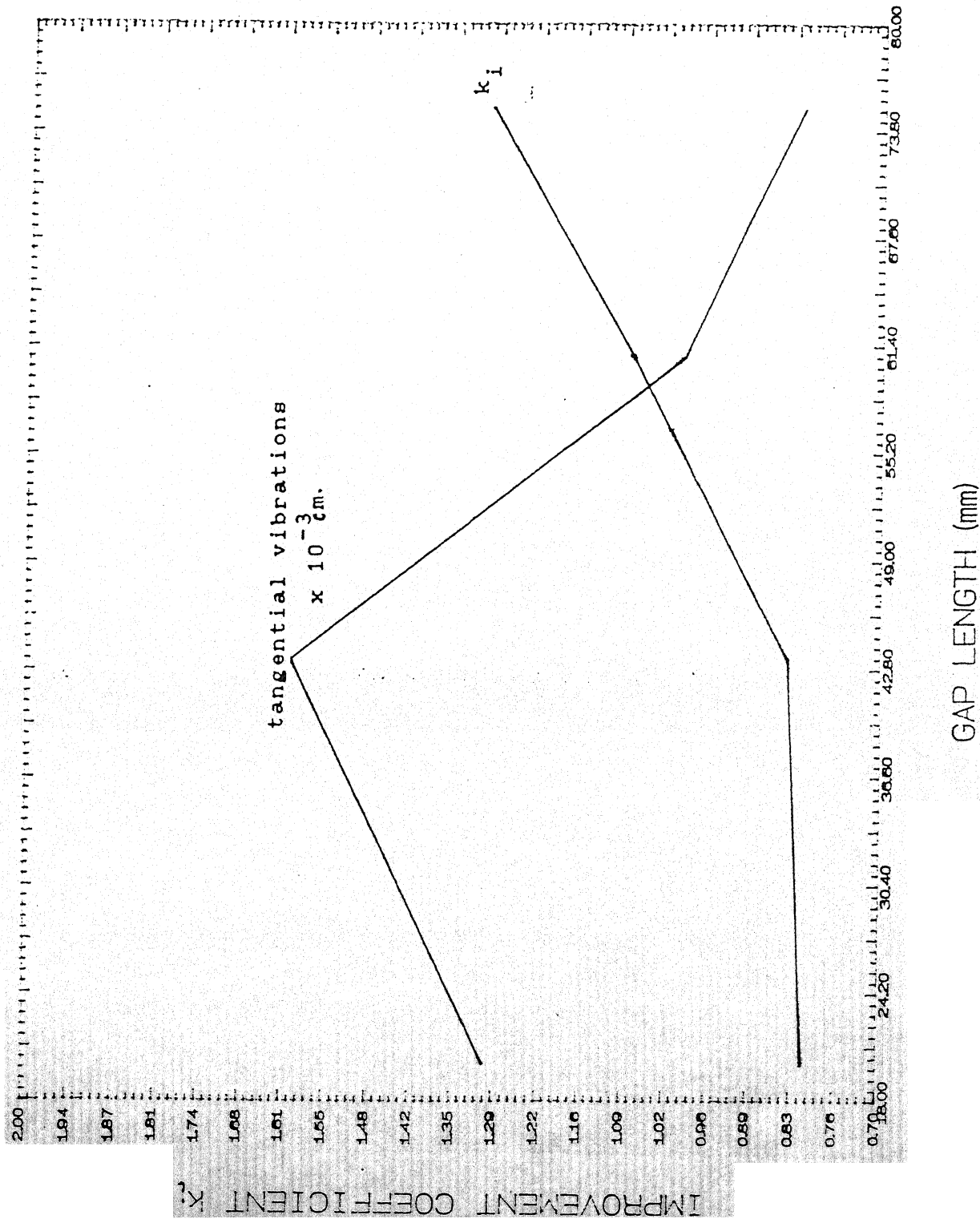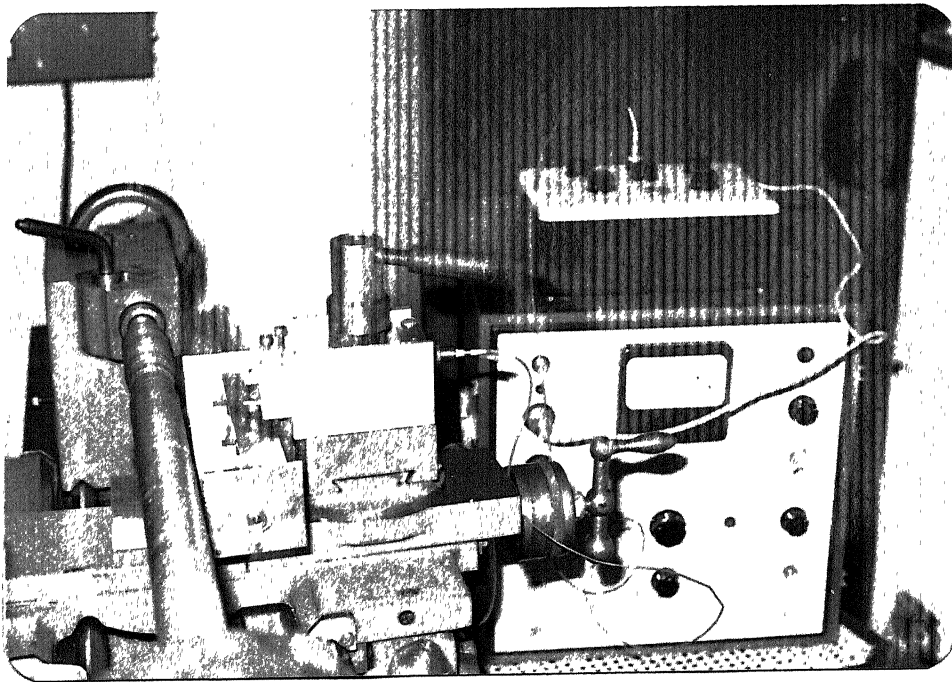
view of the experimental setup

With these conditions in mind, a flat mild steel plate 3 mm thick and 32 mm wide is used as an absorber.

b) Tool Holder:

The tool holder has to satisfy certain requirements. It should be able to withstand all the forces acting on the tool. There must be a provision for the excitation forces to be transmitted to the tool. However, the tool holder must not be too weak so as to deflect excessively under the action of the cutting forces. Furthermore, arrangements also have to be made for proper placing of the sensor to sense the vibration.

The tool holding block, (Fig. 3.1) made of aluminium, is supported about the central rod of the conventional tool post base and supported on a stiff spring to prevent complete contact with the base. To achieve this, a groove, having the diameter of the spring, is cut underneath the block and the block is clamped from the top by a clamping nut. The tool holder is also supported at the two sides. To accommodate the tool in the holder, an attachment made of brass (6) is used. The circular attachment has a square hole to accommodate the tool.

3.2 CALIBRATION OF THE ABSORBER

The calibration of the absorber is necessary to determine the stiffness of the absorber as a function of the gap length. This helps to obtain the proper stiffness depending upon the cutting conditions. The calibration is done by setting a particular gap

length between the supports and then applying a predetermined load. From the load deflection curve (Fig. 3.2) it is possible to know the stiffness of the spring as a function of the gap length.

In order to set,measure and record the signals under actual machining conditions, the following instruments are used:

a)   vibration pickup amplifier, (preamplifier)

b)   microphone amplifier.

Vibration signals sensed by piezoelectric transducers are fed through the preamplifier to the microphone amplifier and the output is recorded. Prior to recording the data, both the preamplifier and the microphone amplifier are calibrated as per instructions of the manual.

CHAPTER 4

EXPERIMENTAL RESULTS AND DISCUSSIONS

A series of experiments is conducted to determine the performance of the absorber system. Experiments are conducted at two feeds and two feeds listed in Table (4.1).

The workpiece, a mild steel rod is clamped in a three-jaw chuck and is supported at the tailstock by revolving centres. The workpiece is first rough turned to remove the ovality as well as the hard scale. This rough turned workpiece is then turned with the tool in the presence of the absorber. Since dimensional accuracy is not required in rough turning operations, therefore for experiments, small values of depth of cut and feed rates are used. Again, since the technological capabilities of the machine is not good enough, it becomes necessary to operate at lower speeds. The depth of cut is adjusted by a dial gauge.

The job is first turned in the absence of absorber upto a certain distance. The absorber is then incorporated and the cutting is commenced. Readings are taken at four gap lengths. While changing the gap length, the machine is stopped and the gap is adjusted by rotating the screw until the proper gap length is reached. The gap lengths are selected arbitrarily for the first set and are kept constant for all the subsequent sets of experiment. The vibrations of the tool holder is recorded in all cases in the microphone amplifier. The voltage level

(peak-to-peak) indicates the improvement.

Smaller the voltage level, better is the performance since the input voltage at the transducer is proportional to the amplitude of vibrations. The values of amplitudes obtained for the cutting conditions under which the experiments are conducted are shown in Table (4.1). Measurements are taken for both tangential and radial directions.

4.2 DISCUSSION OF RESULTS

From Table 4.1 it becomes evident that the present absorber system is capable of improving the vibration characterisdtics under the cutting conditions used.

The improvement in surface finish varied under the various cutting conditions. This improvement can be expressed by a coefficient defined as improved coefficient (k_i)
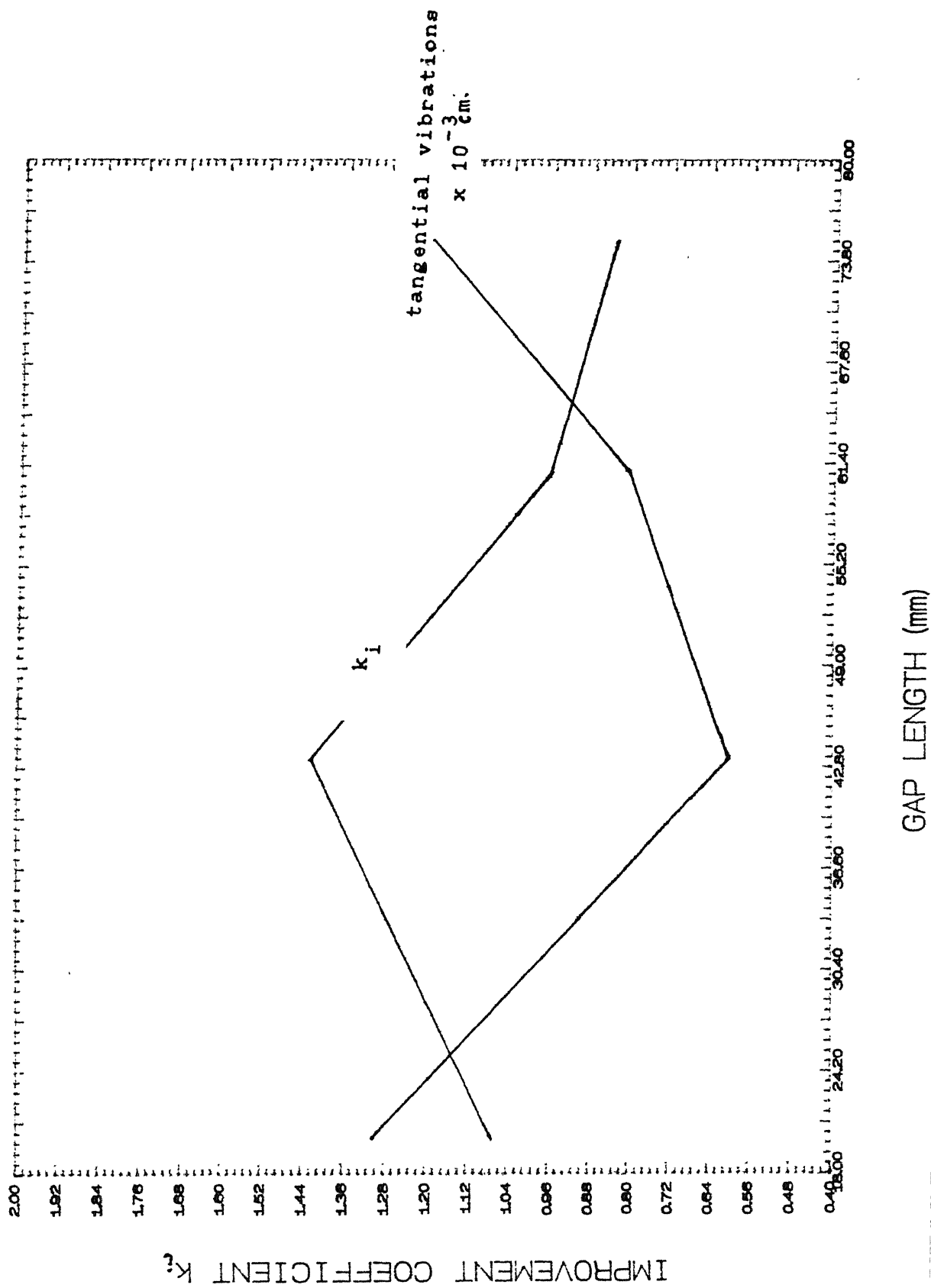
$$= \frac{\text{peak value of displacement without absorber}}{\text{peak value with absorber}}$$

The values of the improvement coefficient for different cutting conditions have been listed in Table 4.2. The values show a reasonable improvement in the range of experiments conducted. Figures (4.1a - 41b) indicate the improvement as a function of the gap length.

Table 4.1

Feed (mm/rev)	r.p.m.	tangential vibrations x 10^{-3} cm.		radial vibrations x 10^{-3} cm.		gap length (mm)
		uncont-rolled	cont-trolled	uncont-rolled	cont-rolled	
0.028	48	1.2	–	3	–	–
			1.3		2.8	20.14
			0.6		2.1	43.34
			0.8		3.1	60.84
			1.2		3.6	75.14
0.113	48	1.4		2.6		
			1.3		3.2	20.14
			1.6		3.1	43.34
			1.0		2.4	60.84
			0.82		2.0	75.14
0.028	182	1.8		2.4		
			1.5		3.0	20.14
			1.6		2.6	43.34
			1.8		2.0	60.84
			1.4		2.4	75.14
0.113	182	2.2		7.8		
			2.2		5.1	20.14
			1.8		5.0	43.34
			2.8		6.0	60.84
			3.8		7.2	75.14

FIG. 4.1 A EXPERIMENTAL RESULT (SET-1)

tangential vibrations
x 10^{-3} cm.

k_i

GAP LENGTH (mm)

IMPROVEMENT COEFFICIENT k_i

2.00
1.92
1.84
1.76
1.68
1.60
1.52
1.44
1.36
1.28
1.20
1.12
1.04
0.96
0.88
0.80
0.72
0.64
0.56
0.48
0.40

18.00
24.20
30.40
36.60
42.80
49.00
55.20
61.40
67.60
73.80
80.00

B EXPERIMENTAL RESULT (SET-2)

tangential vibrations x 10⁻³ cm.

k_i

GAP LENGTH (mm)

IMPROVEMENT COEFFICIENT k_i

'c EXPERIMENTAL RESULT (SET-3)

tangential vibrations
x 10⁻³ cm.

k_i

GAP LENGTH (mm)

IMPROVEMENT COEFFICIENT k_i'

2.00
1.93
1.86
1.79
1.72
1.65
1.58
1.51
1.44
1.37
1.30
1.23
1.16
1.09
1.02
0.95
0.88
0.81
0.74
0.67

18.00 24.20 30.40 36.60 42.80 49.00 55.20 61.40 67.60 73.80 80.00

EXPERIMENTAL RESULT (SET-4)

tangential vibrations
x 10^{-3} cm.

k_i

GAP LENGTH (mm)

IMPROVEMENT COEFFICIENT k_i

4.3 CONCLUSIONS

A passive absorber for controlling chatter is lathe tools has been designed based on the principles of dynamic vibration absorbers and tested experimentally. The main aim is to improve the stability limit resulting in improved surface conditions of the workpiece.

From a comparison of the theoretical and actual results it can be seen that the predictions of the theory tally with the experiments as far as the nature of predictions is concerned. For a given cutting condition, there exists a particular tuning at which the absorber effectiveness is a maximum is predicted by the theory and this has been achieved for all the cutting conditions during experiments. However, since the exact values of the system parameters (i.e. stiffness of the tool holder in the principal direction, natural frequencies, damping etc.) are not known, it is impossible to predict accurately the effective dynamic cutting stiffness for a given cutting condition and tally it with the experimental values.

Significant improvement could not be achieved experimentally for which the possible causes may be traced to a) sliding friction at the sides of tool holder; b) improper clamping of the absorber spring; c) preloading effects on the springs. Any effort, in future, should tackle properly the above stated problems.

4.4 SCOPE OF FUTURE WORK

The present work is just the first step in trying to design an absorber to control chatter. This design, however should be improved so as to make it possible to be used under practical conditions. To obtain better effectiveness of the absorber, one should take steps to eliminate all possible cases of sliding friction at the sides. For this, roller bearings should be used at the side plates. Hardened steel plates or spring steel can be used to replace the ordinary mild steel plates used during experiments to avoid preloading effects. It is expected that, if properly designed, the present system can have the potential to improve upon the existing tool post for the conventional lathes. However, to achieve, this goal a huge amount of experiments under varying cutting conditions have to be conducted.

A major drawback of the system is that the absorber is of the passive type and, hence, has to be properly turned for every set of experiments to achieve the maximum effectiveness. In order to make it capable of controlling chatter over a wide range of frequencies, the present system can be converted into an active control one. For this, a sensor may be incorporated into the tool holder which after sensing the chatter frequency will feed it into a servosystem. The servosystem through aproper logic will automatically adjust the gap between the spring supports so as to obtain the highest stability value. However, this will introduce transient effects caused by the inertia of the servosystem into the main system which has to be tackled properly. Thus one can

see, that there exists a great potential, both from the
experimental and theoretical point of view to develop the present
system.

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APPENDIX

SPECIFICATION OF MEASURING INSTRUMENTS

a)   <u>Vibration pickup preamplifier Type 1606</u>

Frequency   Range: 0.2   Hz   to   100   kHz   within   ±   0.5dB   when
                         "Sensitivity   Adjustment"   is   set   to   minimum
                         amplification,

                         2 Hz to   20 kHz within ± 3dB   when   "Sensitivity
                         Adjustment: is set to maximum amplification,

Amplification: Maximum 38dB approx. voltage amplification   when
                         set for acceleration measurements.

Input Impedance: 200 M Ω paralleled by approx. 50 pF.

Input Attenuation:"Attenuation":   Two   steps   of   40   dB   with   an
                         accuracy of ±0.5 db rc. position "1".

Mea.output   :   In condition "Acceleration":
voltage   20V peak when "Sensitivity Adjustment" is set to
                         max. amplification

                         10V peak when "Sensitivity Adjustment" is set to
                         min. amplification.

Power Supply   :   100 - 240V without adjustments.

b)   <u>Microphone Amplifier Type 2603</u>

Frequency characteristics: 2Hz to 40 Hz to within ±0.5dB relative
                         to 1 kHz.

                         5 Hz to 20 kHz to within ±0.3dB   relative   to   1
                         kHz

Sensitivity :   Maximum   100 μV   and   minimum   1000V   for   full
                         deflection on the indicating meter.

Input Impedances: 1) Input potentiometer approximately   0.7-1M   Ω,
                         the parallel capacity being   dependent   upon
                         setting

                  2)   Amplifier Input: 2.2 M Ω parallel with 30 pF

3) Condensor microphone: 270 or 700M Ω parallel with 2.5-3 pF when one of the B&K cathode followers is incorporated

Attenuation : Meter Range : Variable in steps of 20dB. Accuracy within ± 0.15dB relative to position "10 mV" and 100 Hz

Range Multi- : Variable in steps of 10 dB. plier Accuracy with ±0.1dB relative to position "X0.01" and 100 Hz

Power Supply : 100-115-127-150-220-240 volts AC 50-400 Hz. Power consumption is approx. 50 watts.

c) Piezoelectric Transducers Type 433A

Reference sensitivity 50 Hzat 27^{o}C and including

Cable capacity of 105 pF.

Voltage sensitivity of 54.6 mV/g

Charge sensitivity 58.9 pC/g

Capacity (including cable) 1078 pF

Maximum transverse sensitivity at 30 Hz 1.8%

Undamped natural frequency 48 kHz.

Date Slip

This book is to be returned on the date last stamped.